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U.S. DEPARTMENT OF COMMERCE / National Bureau of Standards

The Thermodynamic Properties of Helium II from 0 K to the Lambda Transitions

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Robert D. McCarty

Thermophysical Properties Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

U.S. DEPARTMENT OF COMMERCE, Philip M. Klutznick, Secretary

Jordan J. Baruch, Assistant Secretary for Productivity, Technology and Innovation

NATIONAL BUREAU OF STANDARDS, Ernest Ambler, Director

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Robert D. McCarty

Thermophysical Properties Division
National Engineering Laboratory
National Bureau of Standards
Boulder, Colorado 80303

The equation of state of He-II is modeled by an equation of state explicit in pressure as a function of density and temperature. The equation of state is divided into three regions of temperature 0 to .8 K; .8 to 1.2 K and 1.2 to the lambda temperature for which similar functional forms are used with different adjustable parameters. The combined functions are valid over the entire PT range of the superfluid and may be used for all classical thermodynamic properties. Comparisons between calculated and experimental data are presented. A computer program for calculation of thermodynamic properties (PVT, isochoric heat capacity, isobaric heat capacity, internal energy, enthalpy, entropy, and velocity of sound) is included.

Key words: Computer program; equation of state; deviation plots; helium II, mathematical model; superfluid; thermodynamic properties.

1. Introduction

A reliable mathematical model of the equation of state of a fluid has become an important tool in nearly all applications of the classical thermodynamic properties. Until now, no such model has been available for superfluid helium II. Because of this situation the NASA-Ames Research Center issued a contract to the National Bureau of Standards in Boulder, Colorado to develop such a model. As a result, a model has been developed which is valid for all of the thermodynamic properties over the entire range of pressure and temperature of the superfluid helium II. It is not valid however for the prediction of the anomalous behavior

of the superfluid transition (the lambda line). The purpose of this report is to present the results of the study conducted for NASA.

2. Survey of the Literature

A search of the world's scientific literature began with a computerized search of about 120,000 articles at the Cryogenic Data Center of the National Bureau of Standards in Boulder, Colorado. This search produced about 250 references. Each of these articles was considered for possible applicable information and leads to articles not revealed in the original computer search. The literature search produced an extensive set of experimental data which covered the entire pressure and temperature range of He II, and in some cases regions of overlap between the data sets.

Two extensive sources of data were found in the literature; the data by Brooks and Donnelly [3] and the data by Maynard [7]. The Brooks and Donnelly [3] reference contains extensive tabulations of thermodynamic properties over the entire PT range of the superfluid phase and is truly a monumental effort on the part of the authors. The Maynard reference [7] also contains tabulations of the thermodynamic properties of the superfluid but only for temperatures of 1.2 K and above. The tabulated properties in these two references are of sufficient quality and quantity to satisfy almost any need for tabular properties. However, neither of the references contain a satisfactory mathematical model of the equation of state which is the primary purpose of this work. For reasons discussed later the data of Brooks and Donnelly [3] were selected as the primary source of data for the least squares estimation procedures. In addition, the 1958 helium temperature scale [2] and the saturated liquid densities by Ker and Taylor [5], were used as input data.

3. Vapor Pressure

An expression for the 1958 helium temperature scale from McCarty [8] was used to predict the vapor pressure of helium between .8 and 2.172 K. The expression is

$$\ell n\, P = \sum_{i=1}^{14} B_i T^{(2-i)} \tag{1}$$

Table 1. Coefficients for Eq. 1 (Vapor Pressure)

B_1	=	- 49.510540356
B_2	=	651.9236417
B_3	=	- 3707.5430856
B_4	=	12880.673491
B_5	=	-30048.545554
B_6	=	49532.267436
B_7	=	-59337.558548
B_8	=	52311.296025
B_9	=	-33950.233134
B_{10}	=	16028.674003
B_{11}	=	- 5354.1038967
B_{12}	=	1199.0301906
B_{13}	=	- 161.46362959
B_{14}	=	9.8811553386

where P is in μm Hg = 133.322 Pa and T is in kelvin. The range of validity for eq (1) is 0.5 to 2.172 K. The coefficients to eq (1) are given in table 1. The vapor pressures in table 7 below 0.8 K are calculated via the thermodynamic

conditions for the coexistence of two phases in equilibrium assuming the ideal gas law for gaseous helium and using the Gibbs free energy (G) of Brooks and Donnelly [3] for the liquid phase. The agreement between the vapor pressures calculated from eq (1) and those calculated as described above between 2.172 and 0.5 K is good, with typical differences of 0.1 percent or less. Table 2 gives a detailed comparison between the 1958 helium scale and pressures calculated via the phase rule using the Brooks and Donnelly [3] data for the saturated liquid and the PVT surface of McCarty [8] for the saturated vapor. The agreement (as shown in table 2) between the 1958 scale and this work is a little surprising since the real gas contribution at 2.0 K on the vapor side amounts to a 3.8 percent of the pressure (i.e., the difference between using an ideal gas equation of state and the one from McCarty). At 1.5 K this real gas contribution has fallen to 1.1 percent and at 0.8 K it has disappeared entirely. Below 1 K the value of the Gibbs energy of the saturated liquid has become so small that it has little effect on the solution and the vapor pressure is determined almost entirely by the choice of L_0, the latent heat of vaporization at 0 K. A value of 59.62 joules/mole was used in the 1958 helium vapor pressure scale and a value of 59.60202 joules/mole was used here. In using the Brooks and Donnelly [3] tables for this calculation, both the tabulated G and the tabulated H-TS were used for comparative purposes. The results are similar but the procedure of using the tabulated G produced results slightly closer to the 58 scale. The G obtained by forming H-TS agrees quite well with the tabulated G down to 0.5 K but begins to differ from the tabulated G below 0.5 K and the two values actually have opposite signs at 0.2 K and below. Since the values of G are so small at these temperatures the resulting vapor pressure is the same for either the - or the + value of G.

Table 2. Comparison of 1958 He Vapor-Pressure-Temperature Scale and Values From This Work.

T	P, bar 1958 Scale (eq 1)	P, bar This Work	%
2.10	0.41906×10^{-1}	0.41780×10^{-1}	0.29
2.00	0.31688×10^{-1}	0.31692×10^{-1}	0.00
1.9	0.23303×10^{-1}	0.23281×10^{-1}	0.09
1.8	0.16618×10^{-1}	0.16616×10^{-1}	0.01
1.7	0.11451×10^{-1}	0.11455×10^{-1}	-0.04
1.6	0.75857×10^{-2}	0.75889×10^{-2}	-0.04
1.5	0.47985×10^{-2}	0.48012×10^{-2}	-0.05
1.4	0.28736×10^{-2}	0.28756×10^{-2}	-0.07
1.3	0.16111×10^{-2}	0.16123×10^{-2}	-0.08
1.2	0.83320×10^{-3}	0.83357×10^{-3}	-0.07
1.1	0.38952×10^{-3}	0.38967×10^{-3}	-0.04
1.0	0.15999×10^{-3}	0.16001×10^{-3}	-0.01
.9	0.55431×10^{-4}	0.55436×10^{-4}	-0.01
.8	0.15258×10^{-4}	0.15258×10^{-4}	0.00
.7	0.30376×10^{-5}	0.30385×10^{-5}	-0.01
.6	0.37487×10^{-6}	0.37496×10^{-6}	-0.02
.5	0.21770×10^{-7}	0.21792×10^{-7}	-0.10
.4		0.34626×10^{-9}	
.3		0.42918×10^{-12}	
.2		0.10086×10^{-17}	
.1		0.48395×10^{-34}	

4. Density of the Saturated Liquid and Gaseous Phases

The density of the saturated liquid phase was taken from Ker and Taylor [5] and is given by

$$V_{\mathrm{sat\ liq}} = \ell_1 + \ell_2 |T - T_\lambda| + \ell_3 |T - T_\lambda| \ln |T - T_\lambda| \qquad (2)$$

Table 3. Coefficients for Eq. 2 (Saturated Liquid)

ℓ_1	=	3.31007
ℓ_2	=	0.00742434913
ℓ_3	=	- 0.0059164737553

where V is in liters/mole, T is in kelvin and the ℓ_1, ℓ_2, ℓ_3, and T_λ are given in table 3.

The density of the saturated gaseous state was calculated using the equation of state from McCarty [8] between 0.8 and 2.172 K. Below 0.8 K the density of the saturated gas was calculated using the vapor pressure (as described in the previous section) and the ideal gas equation of state.

5. The Mathematical Model of the Equation of State

It is desirable to model the equation of state of a fluid with a single mathematical function. A single function insures thermodynamic consistency and eliminates potential mathematical continuity problems going from one function to another. Unfortunately a single function for the helium IV equation of state was not found; instead it was necessary to divide the surface into three regions: one from 0.0 to 0.8 K (Region I), one from 0.8 to 1.2 K (Region II), and from 1.2 to the lambda line (Region III). Although mathematical continuity is not achieved at the boundaries, the fit is good enough at the boundaries of each

region to minimize the thermodynamic inconsistencies. For the region of 0 to 0.8 K the function is

$$P = [P_0 (\Delta\rho) + F_s (\Delta\rho,T)]\ 1.01325 \tag{3}$$

where

$$P_0 (\Delta\rho) = \sum_{i=1}^{3} a_i \Delta\rho^i \tag{4}$$

and

$$\begin{aligned} F_s(\Delta\rho,T) = & - F_{S,1}D^2T^4/4 - F_{S,2}D^2T^5/5 - F_{S,3}D^2T^6/6 \\ & - 2F_{S,4}(\Delta\rho)D^2T^3/3 - 2F_{S,5}(\Delta\rho)D^2T^4/4 \\ & - 2F_{S,6}(\Delta\rho)D^2T^5/5 - 2F_{S,7}(\Delta\rho)D^2T^6/6 - 2F_{S,8}(\Delta\rho)D^2T^7/7 \\ & - 3F_{S,9}(\Delta\rho)^2D^2T^3/3 - 3F_{S,10}(\Delta\rho)^2D^2T^4/4 - 3F_{S,11}(\Delta\rho)^2D^2T^5/5 \\ & - 3F_{S,12}(\Delta\rho)^2D^2T^6/6 - 3F_{S,13}(\Delta\rho)^2D^2T^7/7 - 4F_{S,14}(\Delta\rho)^3D^2T^3/3 \\ & - 4F_{S,15}(\Delta\rho)^3D^2T^5/5 - 4F_{S,16}(\Delta\rho)^3D^2T^7/7 - 4F_{S,17}(\Delta\rho)^3D^2T^9/9 \\ & - 4F_{S,18}(\Delta\rho)^3D^2T^{11}/11 - 5F_{S,19}(\Delta\rho)^4D^2T^3/3 \\ & - 5F_{S,20}(\Delta\rho)^4D^2T^5/5 - 5F_{S,21}(\Delta\rho)^4D^2T^7/7 \\ & - 5F_{S,22}(\Delta\rho)^4D^2T^9/9 - 5F_{S,23}(\Delta\rho)^4D^2T^{11}/11 \end{aligned} \tag{5}$$

where P is the pressure in bar, D is the density in moles/liter, T is temperature in kelvin and $\Delta\rho$ = D -36.27877. The coefficients for Region I are given in table 4.

Table 4. Coefficients to Equations for Region I (0 to 0.8 K).

Coefficients for eq 4 (Region I)

(note $\Delta\rho$ for Region I is defined as

D - 36.27877)

a_1	=	2.281877372
a_2	=	0.16820886
a_3	=	0.005277884968

Coefficients for eq 5 (Region I, 0 to 0.8 K)

$F_{s,1}$	=	$-.776003592103 \times 10^{-4}$
$F_{s,2}$	=	$.516985343553 \times 10^{-4}$
$F_{s,3}$	=	$-.185460414352 \times 10^{-5}$
$F_{s,4}$	=	$.993150555179 \times 10^{-6}$
$F_{s,5}$	=	$-.372729528003 \times 10^{-5}$
$F_{s,6}$	=	$.905240314118 \times 10^{-4}$
$F_{s,7}$	=	$-.263138088468 \times 10^{-3}$
$F_{s,8}$	=	$.210133644446 \times 10^{-3}$
$F_{s,9}$	=	$-.251675888508 \times 10^{-6}$
$F_{s,10}$	=	$.246805662352 \times 10^{-5}$
$F_{s,11}$	=	$-.235766906295 \times 10^{-4}$
$F_{s,12}$	=	$.636877273619 \times 10^{-4}$
$F_{s,13}$	=	$-.464000281660 \times 10^{-4}$
$F_{s,14}$	=	$.133504455025 \times 10^{-7}$
$F_{s,15}$	=	$.441252121325 \times 10^{-6}$
$F_{s,16}$	=	$-.390205136440 \times 10^{-5}$
$F_{s,17}$	=	$.569946840678 \times 10^{-5}$
$F_{s,18}$	=	$-.219762939629 \times 10^{-5}$

Table 4. Continued

$F_{s,19}$	=	$-\ .581270462264 \times 10^{-9}$
$F_{s,20}$	=	$-\ .191711245461 \times 10^{-7}$
$F_{s,21}$	=	$.144897551106 \times 10^{-6}$
$F_{s,22}$	=	$-\ .793219612515 \times 10^{-7}$
$F_{s,23}$	=	$-\ .390940913081 \times 10^{-7}$

The model for Regions II and III is given by

$$P = [VP(T) + P_o(\Delta D) + P_r(\Delta D,T)]\ 1.01325 \qquad (6)$$

where P is pressure in bar, D is density in moles/liter, T is temperature in kelvin and VP(T) is given by eq (1). The $P_o(\Delta D)$ is of the same functional form as eq (4) but with different adjustable parameters and a new definition of the independent variable which is $\Delta D = D - D_{sat\ liq}$. D is as before the density in moles/liter and $D_{sat\ liq}$ is $1/V_{sat\ liq}$ which is given by eq (2). The new a_i are given in table 5. The $P_r(\Delta D,T)$ is defined as

$$P_r\ (\Delta D,T) = f_p\ (\Delta D,T) + f_s\ (\Delta\rho,T) \qquad (7)$$

where

$$\begin{aligned} f_p(\Delta D,T) = {} & F_{P,1}\Delta D^3T^3 + F_{P,2}\Delta D^3T^{2.5} + F_{P,3}\Delta D^3T^2 \\ & + F_{P,4}\Delta D^2T^3 + F_{P,5}\Delta D^2T^2 \\ & + F_{P,6}\Delta DT^3 + F_{P,7}\Delta DT^{2.5} + F_{P,8}\Delta DT^2 \end{aligned} \qquad (8)$$

The ΔD and T are as above and the $F_{P,i}$ are given in table 5.

The $f_s(\Delta\rho,T)$ in eq (7) is of the same functional form as eq (5) with different $F_{S,i}$. All of the parameters for Regions II and III are given in table 5.

Table 5. Coefficients to Equations for Regions II and III

Coefficients for eq 4 (Regions II and III, 0.8 to T_λ)

a_1 = 2.241456

a_2 = 0.1757482

a_3 = 0.00470035

Coefficients to eq 5 (Region III, 1.2 to T_λ)

$F_{s,1} = -.299775895293 \times 10^{-3}$

$F_{s,2} = .261528116001 \times 10^{-3}$

$F_{s,3} = -.451073420829 \times 10^{-4}$

$F_{s,4} = -.179926805218 \times 10^{-3}$

$F_{s,5} = .268760818966 \times 10^{-3}$

$F_{s,6} = .153832317516 \times 10^{-4}$

$F_{s,7} = -.162726148595 \times 10^{-3}$

$F_{s,8} = .477756675722 \times 10^{-4}$

$F_{s,9} = -.356060361531 \times 10^{-4}$

$F_{s,10} = .407625370109 \times 10^{-3}$

$F_{s,11} = -.713769173335 \times 10^{-3}$

$F_{s,12} = .449456804718 \times 10^{-3}$

$F_{s,13} = -.913635541095 \times 10^{-4}$

$F_{s,14} = -.975555037829 \times 10^{-5}$

$F_{s,15} = .121659779679 \times 10^{-4}$

$F_{s,16} = -.528306039117 \times 10^{-5}$

Table 5. Continued

$$F_{s,17} = .311573112016 \times 10^{-6}$$
$$F_{s,18} = .613299771434 \times 10^{-7}$$
$$F_{s,19} = .506000325098 \times 10^{-6}$$
$$F_{s,20} = -.612590386700 \times 10^{-6}$$
$$F_{s,21} = .230922759488 \times 10^{-6}$$
$$F_{s,22} = .327499222785 \times 10^{-8}$$
$$F_{s,23} = -.515534867647 \times 10^{-8}$$

Coefficients for eq 5 (Region II, 0.8 to 1.2 K)

$$F_{s,1} = .108660418499 \times 10^{-2}$$
$$F_{s,2} = -.217871751436 \times 10^{-2}$$
$$F_{s,3} = .102911648479 \times 10^{-2}$$
$$F_{s,4} = .189253572751 \times 10^{-2}$$
$$F_{s,5} = -.674364748289 \times 10^{-2}$$
$$F_{s,6} = .798926642309 \times 10^{-2}$$
$$F_{s,7} = -.344107467055 \times 10^{-2}$$
$$F_{s,8} = .299781633163 \times 10^{-3}$$
$$F_{s,9} = -.214084674667 \times 10^{-3}$$
$$F_{s,10} = .335439600940 \times 10^{-3}$$
$$F_{s,11} = .559092792724 \times 10^{-3}$$
$$F_{s,12} = -.119903558078 \times 10^{-2}$$

Table 5. Continued

$F_{s,13} = .526331681180 \times 10^{-3}$

$F_{s,14} = .118775501632 \times 10^{-4}$

$F_{s,15} = -.459408808154 \times 10^{-4}$

$F_{s,16} = .519701003921 \times 10^{-4}$

$F_{s,17} = -.196070771338 \times 10^{-4}$

$F_{s,18} = .731453369826 \times 10^{-6}$

$F_{s,19} = -.526985760908 \times 10^{-6}$

$F_{s,20} = .169561251135 \times 10^{-5}$

$F_{s,21} = -.131795348291 \times 10^{-5}$

$F_{s,22} = -.714287537326 \times 10^{-7}$

$F_{s,23} = .258759130915 \times 10^{-6}$

Coefficients for eq 9 (Regions II and III, 0.8 to T_λ)

$F_{p,1} = -.320783527549$

$F_{p,2} = .580145141306$

$F_{p,3} = -.294344361744$

$F_{p,4} = .290449403103$

$F_{p,5} = .801446582474$

$F_{p,6} = -.175703015761$

$F_{p,7} = .400129303603$

$F_{p,8} = -.255176262894$

6. The Derived Thermodynamic Properties

All of the He II property values tabulated here have been calculated using the equation of state, as described in the previous sections, applied to the following relationships

$$\Delta S = -\int_{\rho_{sat}}^{\rho} \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial T}\right)_\rho d\rho \tag{9}$$

$$\Delta H = \int_{\rho_{sat}}^{\rho} \left[\frac{P}{\rho^2} - \frac{T}{\rho^2} \left(\frac{\partial P}{\partial T}\right)_\rho\right] d\rho + P/\rho \tag{10}$$

$$\Delta C_v = -T \int_{\rho_{sat}}^{\rho} \frac{1}{\rho^2} \left(\frac{\partial^2 P}{\partial T^2}\right)_\rho d\rho \tag{11}$$

$$C_p = C_v + \left(\frac{\partial P}{\partial T}\right)^2 / \left(\frac{\partial P}{\partial \rho}\right)_T \frac{1}{\rho^2} \tag{12}$$

$$W = \left[\left(\frac{C_p}{C_v}\right) \left(\frac{\partial P}{\partial \rho}\right)_T\right]^{1/2} \tag{13}$$

The S, H and C_v of the saturated liquid state were taken from Brooks and Donnelly [3]. The tabulated reference values of Brooks and Donnelly [3] and those of Maynard [7] must be adjusted to be consistent with the published tables of McCarty [8]. The adjustment of 59.869851 joules/mole must be added to the enthalpy. The saturated vapor values tabulated here were calculated using the equation of state by McCarty. Computer program listings for all of the He-II properties are given in Appendix A.

7. Computer Programs

The listings of the computer programs for the properties of He-II in Appendix A are the programs used to calculate the various thermodynamic properties in tables 7 and 8. In general the Fortran functions have been named using the first letter or two to describe the property and the last three letters to describe the fluid. For example FUNCTION CVHE2(D,T) is the routine for specific

heat capacity at constant volume and the fluid is He-II. In addition to the subprograms used in the example in Appendix A, there are a few other routines which are not used in the calculation of the properties but have been included here for the reader's convenience. These routines are PMELT2(T), FINDTD(D), FINDTP(P), DENLAM(T) and PRSLAM(T). The PMELT2(T) routine calculates the melting pressure of He-II according to the work of Grilly [4]. The other four routines mentioned above give the PVT of the lambda line. FINDTP(P) calculates the temperature given a pressure; PRSLAM(T) calculates the pressure given a temperature; DENLAM(T) calculates the density given a temperature; and FINDTD(D) calculates the temperature given a density. The PVT relations of the lambda line are from the work of Kierstead [6]. The remainder of the routines are used in the calculation of the He-II properties.

8. Estimated Accuracy of the Thermodynamic Properties

The accuracy of the thermodynamic properties in tables 7 and 8 depends almost entirely upon the accuracy of the data used in developing the mathematical models. Figures 1 through 9 illustrate the deviations between properties calculated using the methods described in this work and data from the two primary sources of Brooks and Donnelly [3] and Maynard [7]. In viewing these deviation plots, it should be kept in mind that the Brooks and Donnelly [3] data were used in the fit, over the entire temperature range but comparisons to the Maynard data are included to illustrate the differences (or agreement) between the two major sources in their range of overlapping pressures and temperatures.

No attempt was made to force the equation of state to reproduce the asymptotic behavior of He-II as the lambda line is approached. However, as may be seen in table 6, with the exception of the specific heat capacities, the performance of the equation of state along the lambda line is comparable with the

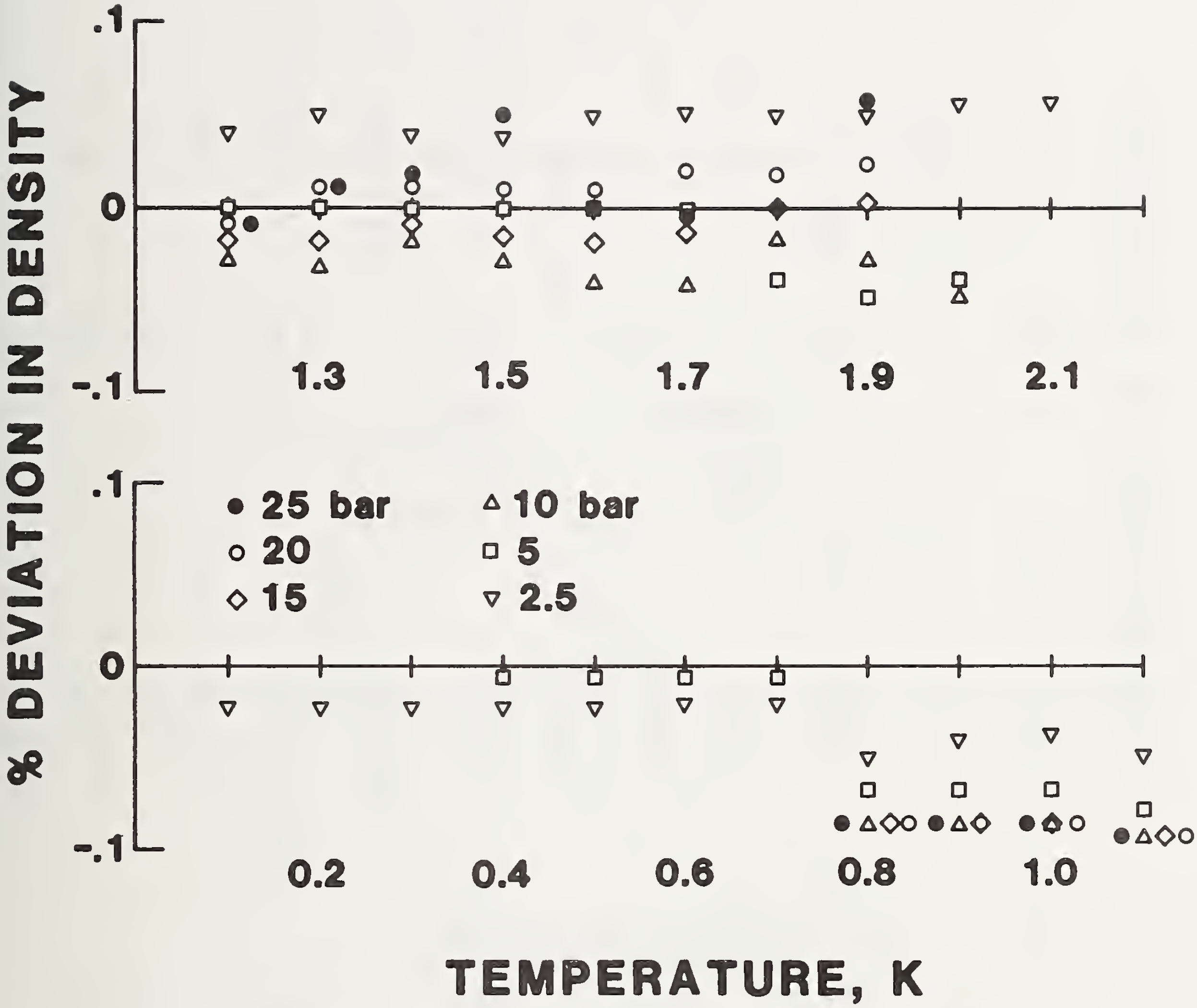

Figure 1. Differences between calculated densities and those of Brooks and Donnelly [3].

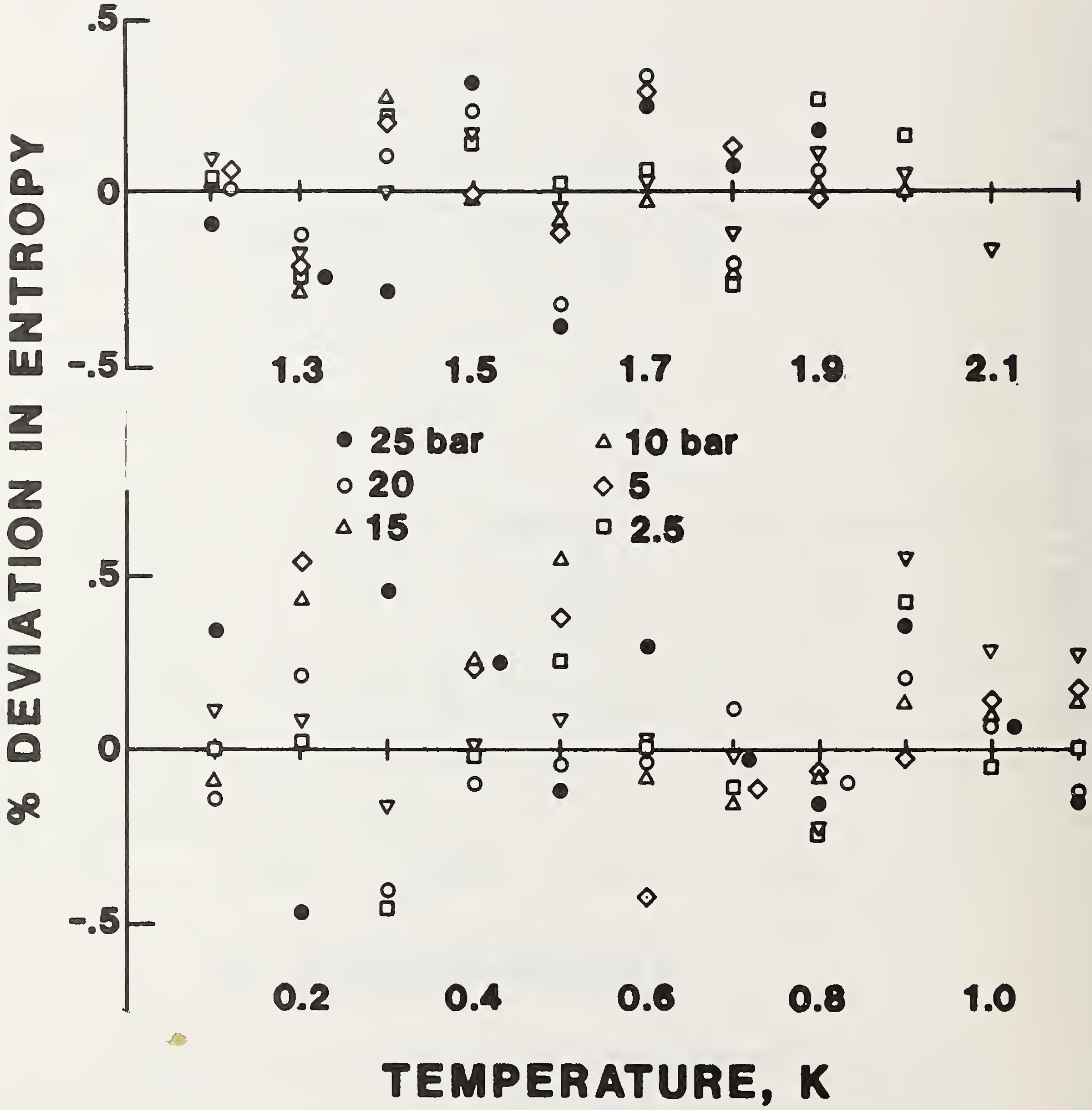

Figure 2. Differences between calculated entropy and those of Brooks and Donnelly [3].

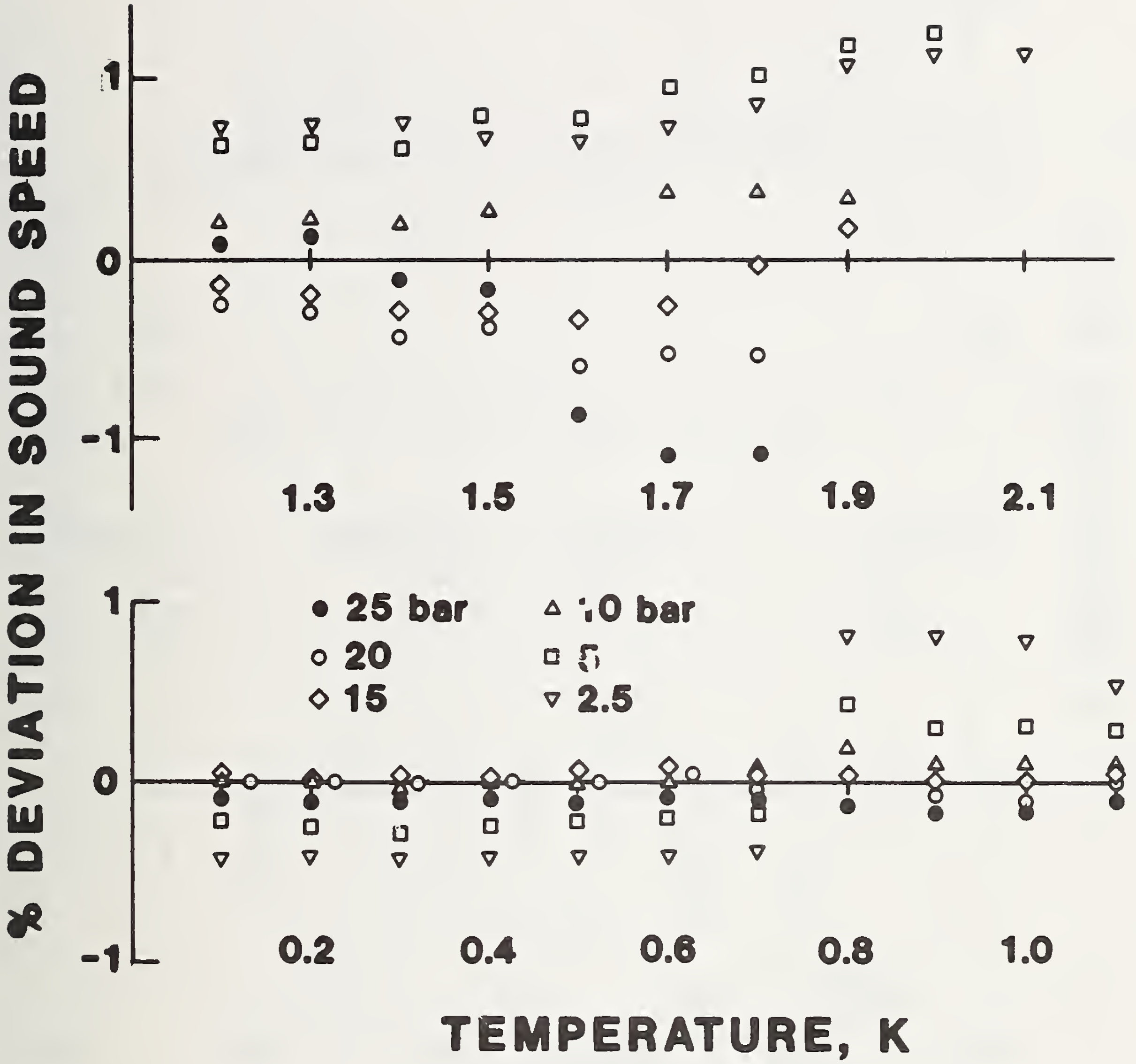

Figure 3. Differences between calculated sound speed and those of Brooks and Donnelly [3].

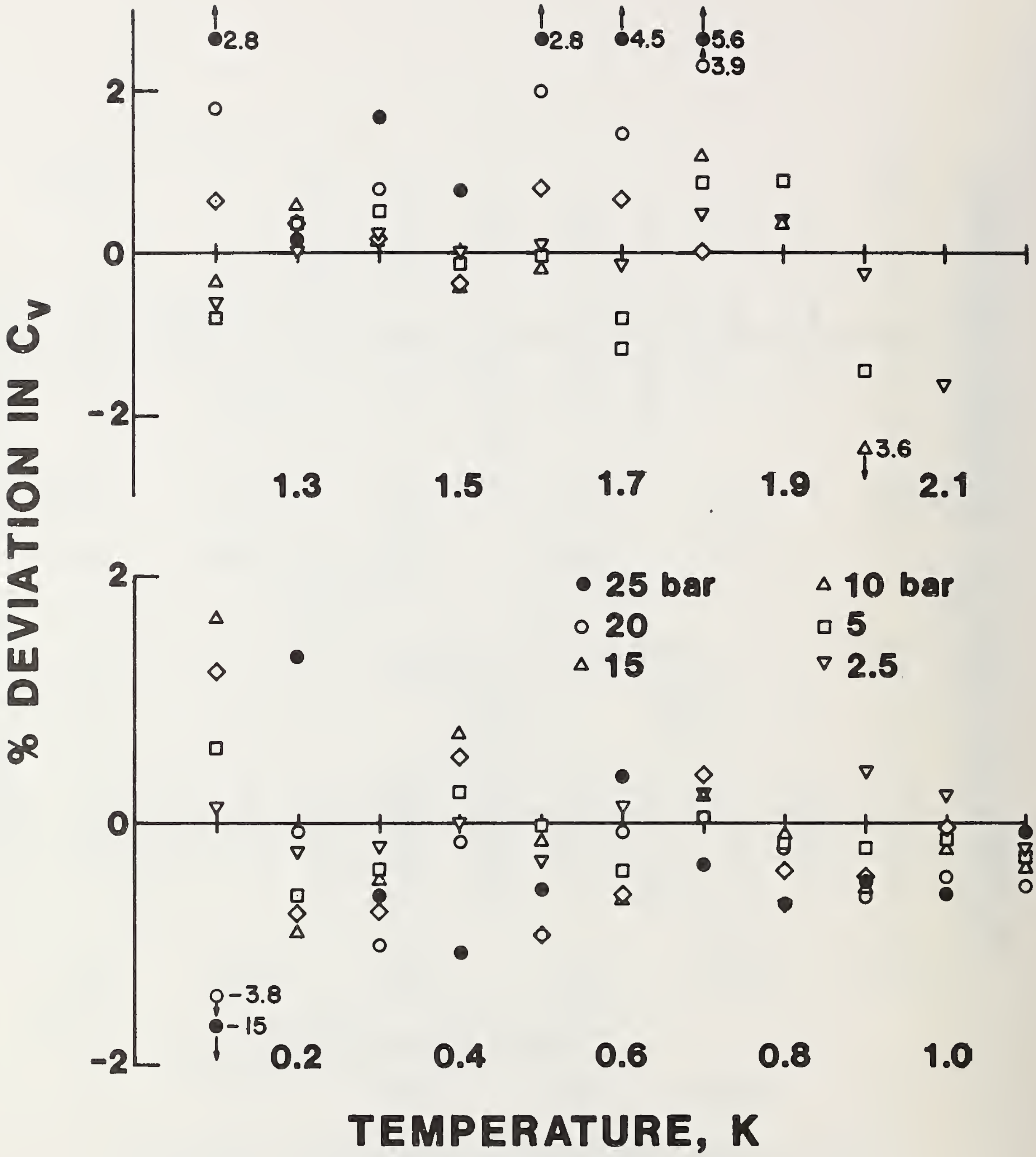

Figure 4. Differences between calculated specific heat capacity at constant value and those of Brooks and Donnelly [3].

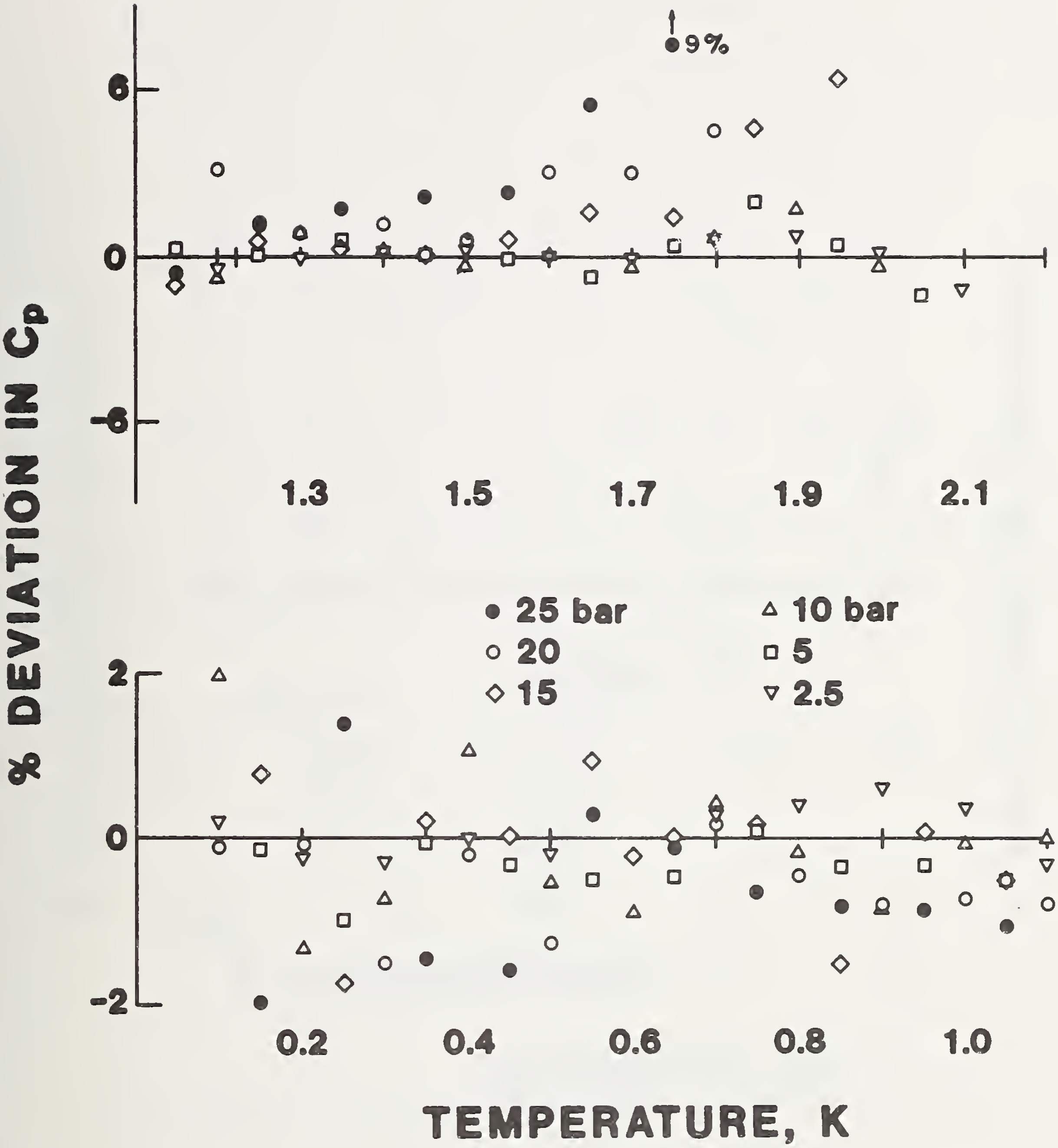

Figure 5. Differences between calculated specific heat capacity at constant pressure and those of Brooks and Donnelly [3].

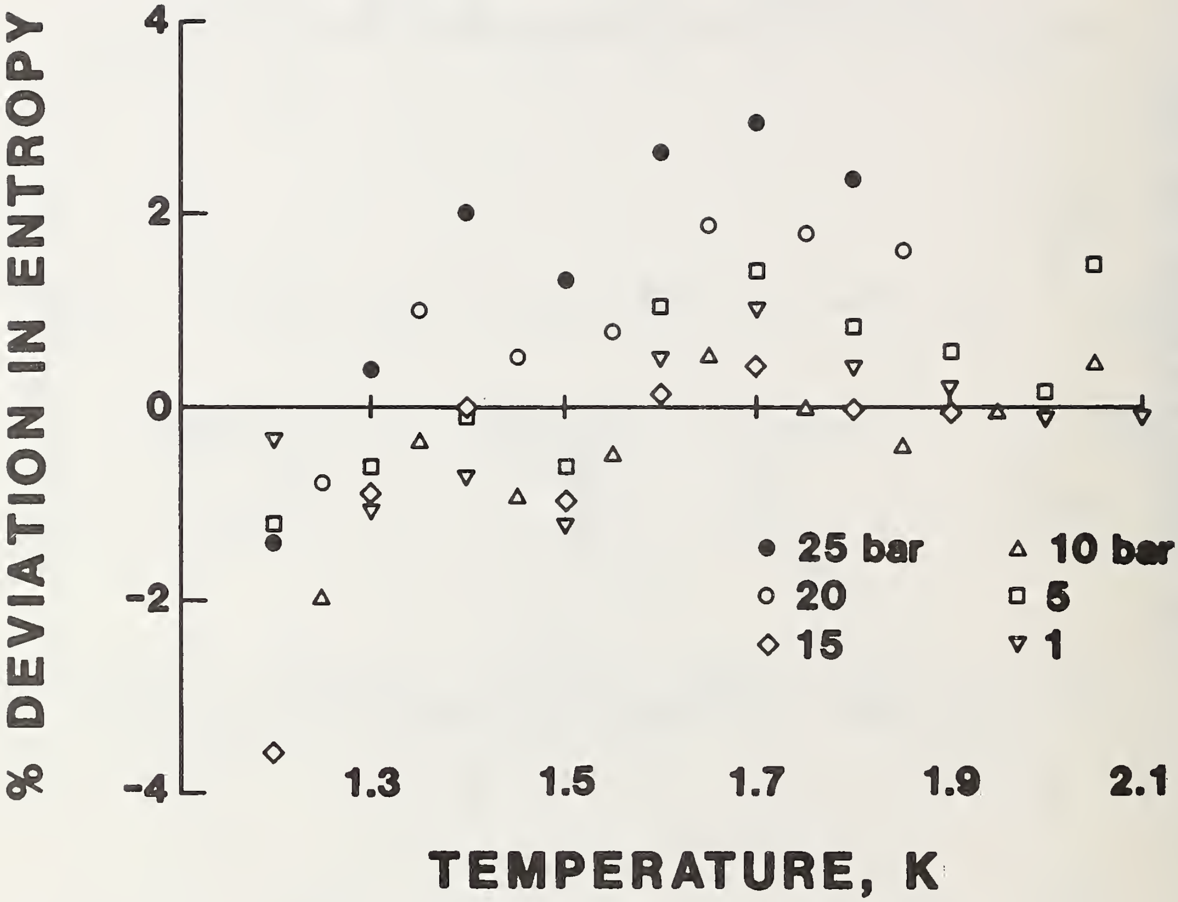

Figure 6. Differences between calculated entropy and those of Maynard [7].

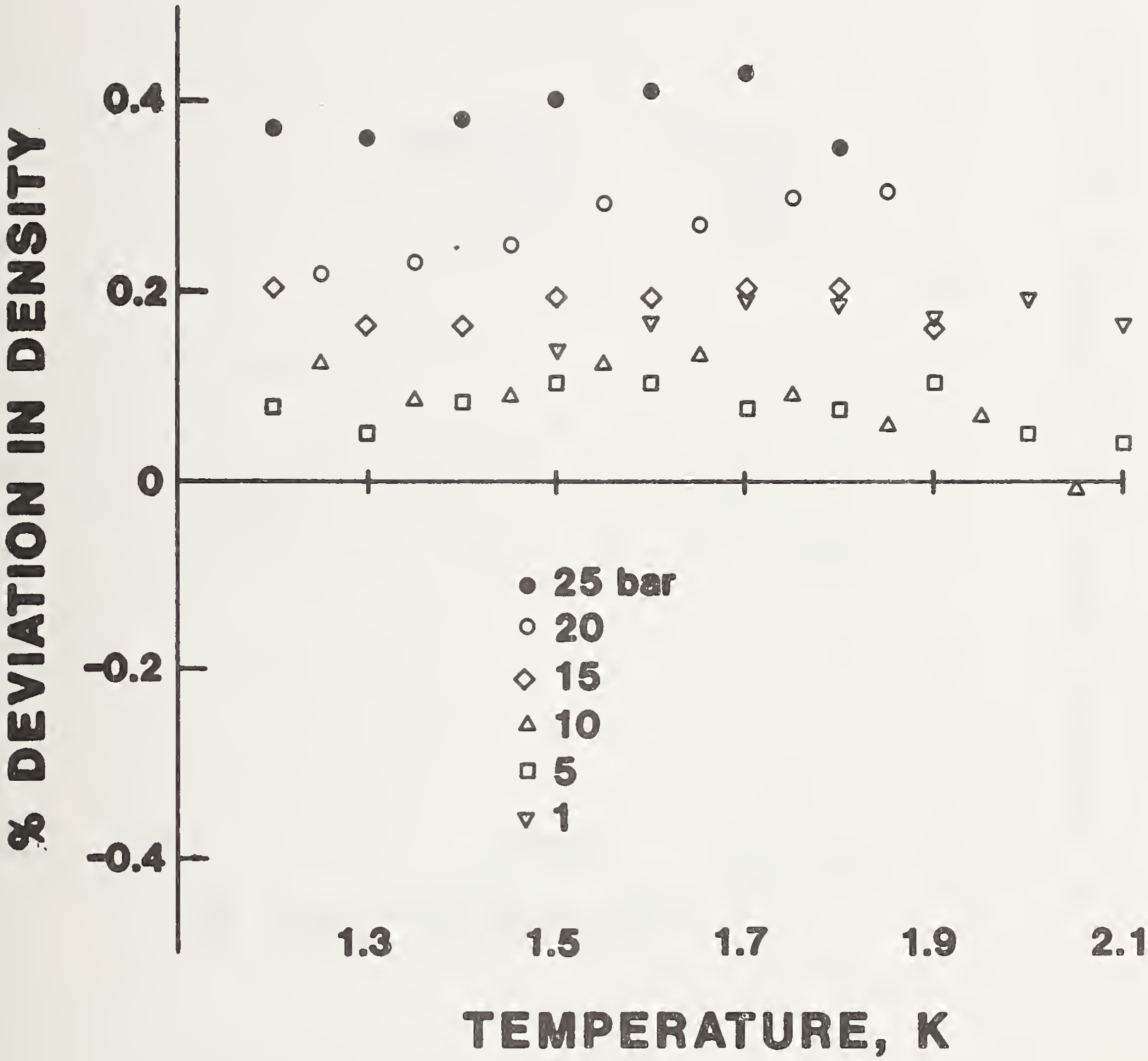

Figure 7. Differences between calculated density and those of Maynard [7].

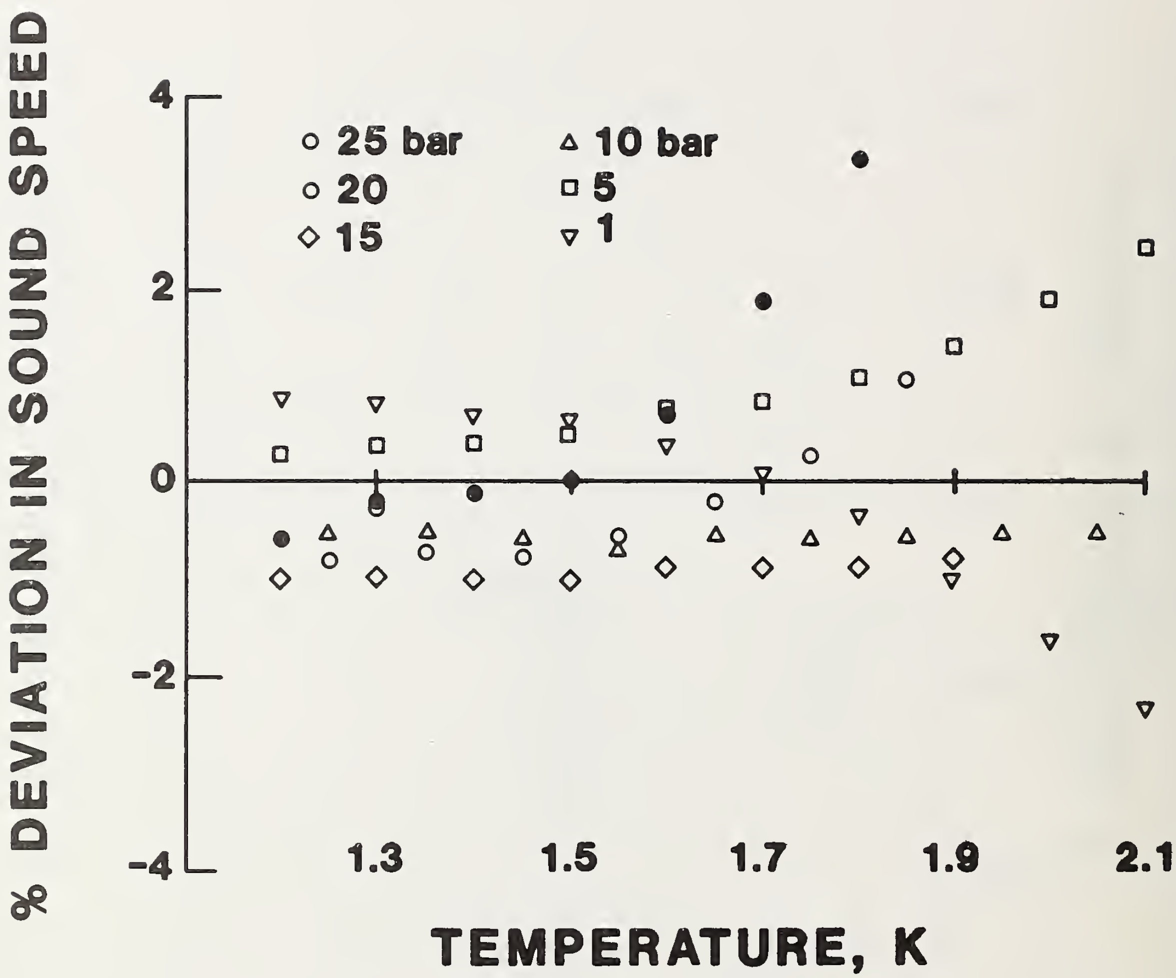

Figure 8. Differences between calculated sound speed and those of Maynard [7].

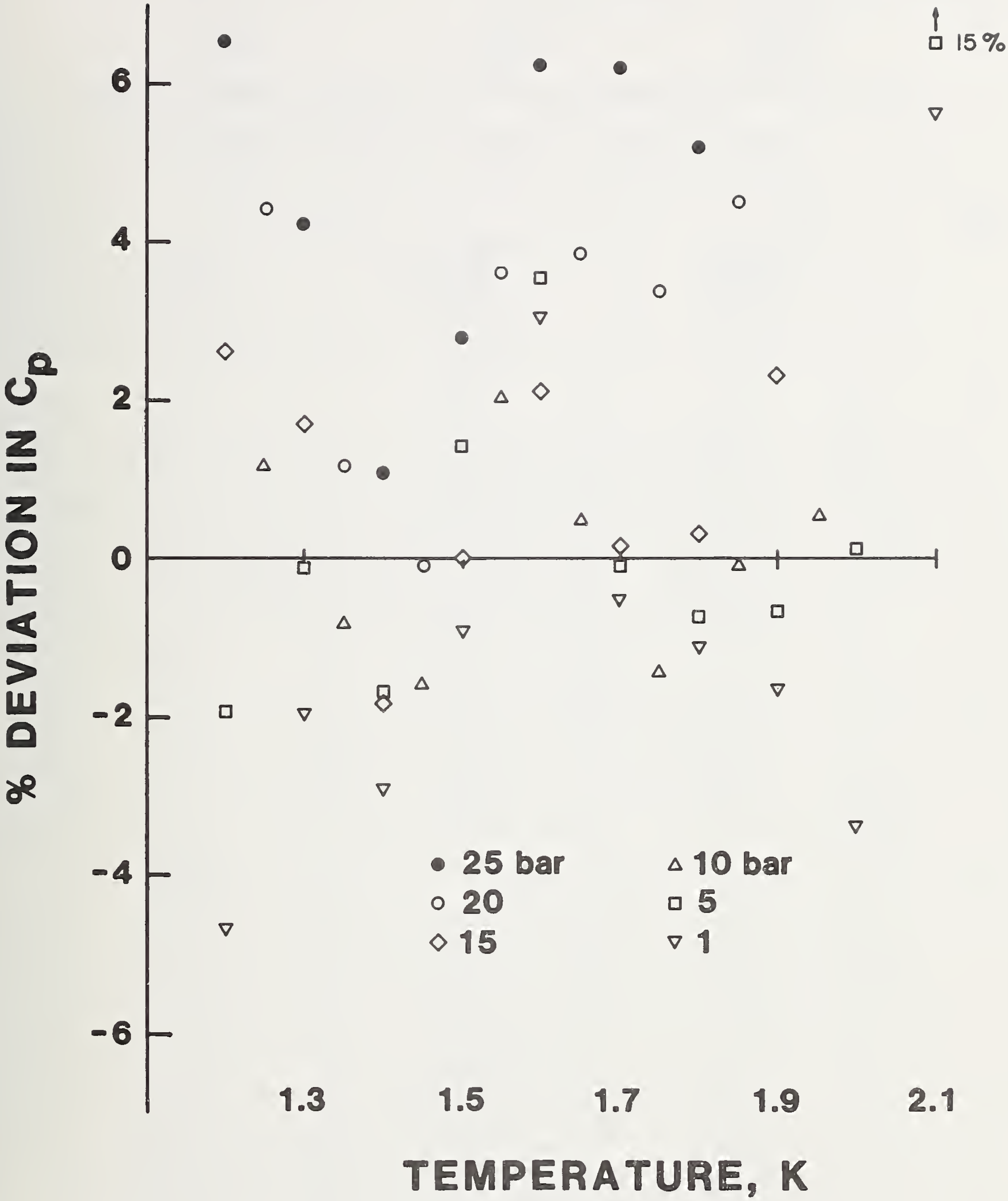

Figure 9. Differences between calculated specific heat capacity at constant pressure and those of Maynard [7].

rest of the surface. The PVT in table 6 are from Kierstead [6] and the entropies and sound velocity are from Ahlers [1].

Table 6. Selected Properties of He Along the Lambda Line With Comparison to the Equation of State in This Work.

P_{bar}	T	ρ_1	%	C_2	$\%_{diff}$	S_2	%
	K	$\frac{\text{moles}}{\text{liter}}$		$\frac{\text{meters}}{\text{second}}$		$\frac{\text{joules}}{\text{mole}}$	
1.	2.163	37.001	- .2	225.3	-3.4	6.1560	1.9
2.	2.154	37.451	- .2	233.4	- .3	6.0719	3.0
3.	2.143	37.900	.2	241.0	2.0	5.9879	4.0
4.	2.133	38.300	.15	248.1	2.5	5.9118	4.7
5.	2.122	38.675	.06	254.8	2.2	5.8398	3.8
6.	2.111	39.050	.07	261.0	1.5	5.7717	3.2
7.	2.099	39.399	0.04	266.9	0.7	5.7037	2.7
8.	2.087	39.724	0.06	272.5	0.1	5.6397	1.9
9.	2.075	40.049	.07	277.7	-0.4	5.5796	1.8
10.	2.063	40.349	.1	282.6	-0.9	5.5196	1.5
11.	2.051	40.649	.1	287.3	-1.2	5.4635	1.2
12.	2.038	40.923	0.15	291.8	-1.3	5.4075	1.3
13.	2.025	41.198	0.16	296.0	-1.2	5.3515	1.2
14.	2.012	41.473	0.2	300.1	-1.0	5.2994	1.3
15.	1.998	41.748	0.3	303.9	-0.7	5.2474	1.6
16.	1.985	41.998	0.3	307.6	-0.2	5.1954	1.6
17.	1.971	42.248	0.3	311.1	0.2	5.1473	1.8
18.	1.957	42.497	+ .3	314.5	0.8	5.0953	1.9
19.	1.942	42.722	- .3	317.7	1.4	5.0473	2.3

Table 6. Continued

P_{bar}	T	ρ_1	%	C_2	$\%_{diff}$	S_2	%
	K	$\frac{\text{moles}}{\text{liter}}$		$\frac{\text{meters}}{\text{second}}$		$\frac{\text{joules}}{\text{mole}}$	
20.	1.928	42.947	0.3	320.7	1.9	4.9992	2.5
21.	1.913	43.172	0.4	323.6	2.5	4.9512	2.2
22.	1.897	43.397	.04	326.3	3.0	4.9032	2.5
23.	1.882	43.622	0.4	328.8	3.5	4.8552	2.3
24.	1.866	43.846	0.4	331.1	3.8	4.8071	2.4
25.	1.850	44.046	0.4	333.3	4.0	4.7591	2.6

When using the equation of state, the best accuracy will result when the input variables are pressure and temperature rather than density and temperature. This is true of all equations of state in the liquid region and is a consequence of the nature of the liquid surface rather than the inadequacies of a mathematical model.

Figures 1 through 9 illustrate a wide variation in the ability of the model to reproduce some of the properties. Below 1.2 K the deviations in general between calculated and input properties are noticably less than those above 1.2 K. The thermodynamic consistency of the data seems to be better at the lower temperatures, and Brooks and Donnelly [3] estimate the accuracy of the data is better at the lower temperatures. At the beginning of the project, Maynard [7] was chosen as the primary source of data above 1.2 K. This decision was based on preliminary analysis of the data themselves and on the claims of accuracy made by the author. During the course of the investigation it became apparent that the Brooks and Donnelly [3] data are the more internally consistent of the two sets and these data were therefore chosen to be used in the fit over the entire

temperature range. The problem with the Maynard [7] data seems to be with the density. Fitting the Maynard data by itself using a simultaneous data fitting technique (which is necessary for this thermodynamic surface) where the PVT and the entropy are used in the least squares estimation of the adjustable parameters, produces a reasonably good fit to those two properties. Comparisons to the other thermodynamic properties such as C_p and velocity of sound with corresponding calculated values indicate large systematic deviations. Further these deviation patterns are the same when comparing with the Brooks and Donnelly data except for PVT, where there is a definite difference of about 0.3 percent between Maynard and Brooks and Donnelly. Performing the same estimation procedure as before (i.e., using PVT and entropy as input) but using the Brooks and Donnelly data instead of the Maynard data produces a better overall fit to both sets of data except again for the PVT of Maynard. This strongly suggests that the Maynard PVT are inconsistent with the true thermodynamic surfaces. Quite a lot of experimentation using various different combinations of the data as input to the least squares estimation procedure indicated several other points. First it is impossible to produce an equation of state for Helium-II which is adequate to calculate all the thermodynamic properties using only PVT data as input to the estimation procedure. This is true because the $(\partial P/\partial T)_\rho$ and $(\partial^2 P/\partial T^2)_\rho$ are so small that several orders of magnitude of precision in the PVT would be required to determine them from the PVT alone. Second using more than two types of data, for example, PVT, entropy and C_v simultaneously, improved the C_v representation (but only marginally) over the case where only PVT and entropy were used, however, the slight improvement of C_v degraded the entropy representation noticably. Below 0.8 kelvin the entropy may be fit independently of the PVT using eq (5) and then simply added to eq (4) without any noticable effect on the representation of pressure.

An indication of the accuracy of the properties calculated from the equation of state may be found by consulting the deviation plots, figs. 1 through 9.

9. Property Tables

The thermodynamic properties for He-II given here in tables 7 and 8, with one exception, have been calculated using the computer programs given in the appendix. The properties of the saturated vapor phase in tables 7 and 8 have been calculated using the equation of state from McCarty [8]. To achieve continuity between this work and the enthalpy of the author's 1973 work, a value of 59.869851 joules/mole must be added to the enthalpy of the 1973 work. No adjustment to the entropy is required.

The tables and equations given here are based on the 1958 helium vapor pressure scale and the temperature adjustment made in the author's previous helium work (1973) has not been made here.

The molecular weight of helium is taken to be 4.0026 and the value of R used is .0831434 liter•bar/mole•kelvin.

Table 7. Properties of Coexisting Liquid and Vapor.

PRESSURE BAR	TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY JOULES/MOLE•K	CV JOULES/MOLE•K	CP JOULES/MOLE•K	SOUND SPEED METERS/SECOND
.4707E-01	2.15	.2777E+00	.1021E+03	.4846E+02	.1278E+02	.2239E+02	83.7
.4707E-01	2.15	.3647E+02	.1023E+02	.5656E+01	.3317E+02	.3329E+02	238.0
.4133E-01	2.10	.2486E+00	.1013E+03	.4910E+02	.1276E+02	.2227E+02	82.9
.4133E-01	2.10	.3643E+02	.8903E+01	.4959E+01	.2657E+02	.2664E+02	238.5
.3605E-01	2.05	.2212E+00	.1005E+03	.4977E+02	.1274E+02	.2214E+02	82.1
.3605E-01	2.05	.3640E+02	.7664E+01	.4369E+01	.2361E+02	.2365E+02	238.7
.3123E-01	2.00	.1957E+00	.9962E+02	.5050E+02	.1272E+02	.2202E+02	81.3
.3123E-01	2.00	.3638E+02	.6518E+01	.3804E+01	.2076E+02	.2080E+02	238.5
.2687E-01	1.95	.1720E+00	.9875E+02	.5126E+02	.1270E+02	.2191E+02	80.4
.2687E-01	1.95	.3635E+02	.5525E+01	.3305E+01	.1805E+02	.1808E+02	238.1
.2295E-01	1.90	.1502E+00	.9787E+02	.5207E+02	.1268E+02	.2180E+02	79.6
.2295E-01	1.90	.3633E+02	.4706E+01	.2887E+01	.1545E+02	.1547E+02	237.6
.1945E-01	1.85	.1303E+00	.9698E+02	.5293E+02	.1266E+02	.2169E+02	78.7
.1945E-01	1.85	.3631E+02	.3985E+01	.2507E+01	.1333E+02	.1335E+02	237.1
.1635E-01	1.80	.1122E+00	.9608E+02	.5384E+02	.1264E+02	.2159E+02	77.8
.1635E-01	1.80	.3630E+02	.3354E+01	.2166E+01	.1171E+02	.1172E+02	236.5
.1363E-01	1.75	.9591E-01	.9517E+02	.5480E+02	.1262E+02	.2150E+02	76.8
.1363E-01	1.75	.3628E+02	.2803E+01	.1860E+01	.1018E+02	.1019E+02	236.0
.1126E-01	1.70	.8131E-01	.9424E+02	.5582E+02	.1260E+02	.2141E+02	75.8
.1126E-01	1.70	.3627E+02	.2318E+01	.1582E+01	.8745E+01	.8753E+01	235.6
.9205E-02	1.65	.6833E-01	.9331E+02	.5690E+02	.1258E+02	.2133E+02	74.8
.9205E-02	1.65	.3626E+02	.1910E+01	.1342E+01	.7395E+01	.7401E+01	235.2
.7449E-02	1.60	.5688E-01	.9236E+02	.5805E+02	.1257E+02	.2126E+02	73.8
.7449E-02	1.60	.3625E+02	.1572E+01	.1137E+01	.6200E+01	.6204E+01	234.9
.5958E-02	1.55	.4686E-01	.9140E+02	.5927E+02	.1255E+02	.2119E+02	72.7
.5958E-02	1.55	.3624E+02	.1290E+01	.9605E+00	.5266E+01	.5268E+01	234.7
.4707E-02	1.50	.3818E-01	.9044E+02	.6057E+02	.1254E+02	.2112E+02	71.6
.4707E-02	1.50	.3623E+02	.1046E+01	.8028E+00	.4532E+01	.4534E+01	234.6
.3667E-02	1.45	.3071E-01	.8946E+02	.6196E+02	.1252E+02	.2107E+02	70.5
.3667E-02	1.45	.3623E+02	.8313E+00	.6591E+00	.3879E+01	.3880E+01	234.5
.2815E-02	1.40	.2438E-01	.8848E+02	.6345E+02	.1251E+02	.2102E+02	69.3
.2815E-02	1.40	.3622E+02	.6483E+00	.5320E+00	.3236E+01	.3236E+01	234.6
.2125E-02	1.35	.1906E-01	.8748E+02	.6505E+02	.1250E+02	.2097E+02	68.1
.2125E-02	1.35	.3622E+02	.5017E+00	.4270E+00	.2629E+01	.2629E+01	234.7
.1576E-02	1.30	.1466E-01	.8648E+02	.6676E+02	.1249E+02	.2093E+02	66.9
.1576E-02	1.30	.3622E+02	.3860E+00	.3411E+00	.2105E+01	.2105E+01	234.8
.1145E-02	1.25	.1106E-01	.8548E+02	.6862E+02	.1248E+02	.2090E+02	65.6
.1145E-02	1.25	.3622E+02	.2916E+00	.2677E+00	.1685E+01	.1685E+01	235.0
.8134E-03	1.20	.8180E-02	.8447E+02	.7062E+02	.1248E+02	.2087E+02	64.3
.8134E-03	1.20	.3622E+02	.2160E+00	.2069E+00	.1340E+01	.1341E+01	235.2
.5635E-03	1.15	.5908E-02	.8345E+02	.7280E+02	.1247E+02	.2085E+02	63.0
.5635E-03	1.15	.3624E+02	.1570E+00	.1572E+00	.1043E+01	.1043E+01	232.0
.3794E-03	1.10	.4156E-02	.8243E+02	.7517E+02	.1247E+02	.2083E+02	61.7
.3794E-03	1.10	.3624E+02	.1113E+00	.1170E+00	.7931E+00	.7932E+00	232.3
.2474E-03	1.05	.2837E-02	.8140E+02	.7777E+02	.1247E+02	.2081E+02	60.3
.2474E-03	1.05	.3624E+02	.7737E-01	.8588E-01	.5864E+00	.5864E+00	232.6
.1554E-03	1.00	.1871E-02	.8037E+02	.8062E+02	.1247E+02	.2080E+02	58.8
.1554E-03	1.00	.3624E+02	.5289E-01	.6228E-01	.4204E+00	.4205E+00	232.8
.9358E-04	.95	.1185E-02	.7934E+02	.8378E+02	.1247E+02	.2079E+02	57.3
.9358E-04	.95	.3624E+02	.3578E-01	.4493E-01	.2802E+00	.2802E+00	233.0
.5365E-04	.90	.7173E-03	.7830E+02	.8728E+02	.1247E+02	.2079E+02	55.8
.5365E-04	.90	.3624E+02	.2401E-01	.3234E-01	.1905E+00	.1905E+00	233.2
.2905E-04	.85	.4111E-03	.7727E+02	.9120E+02	.1247E+02	.2079E+02	54.2
.2905E-04	.85	.3624E+02	.1611E-01	.2341E-01	.1271E+00	.1271E+00	233.2
.1526E-04	.80	.2294E-03	.7623E+02	.9529E+02	.2079E+02	.2910E+02	48.2
.1526E-04	.80	.3624E+02	.1085E-01	.1709E-01	.8428E-01	.8431E-01	233.3
.7145E-05	.75	.1146E-03	.7519E+02	.1003E+03	.2079E+02	.2910E+02	46.7
.7145E-05	.75	.3628E+02	.7223E-02	.1260E-01	.5521E-01	.5523E-01	240.3
.3038E-05	.70	.5220E-04	.7415E+02	.1059E+03	.2079E+02	.2910E+02	45.1
.3038E-05	.70	.3628E+02	.4973E-02	.9495E-02	.3659E-01	.3660E-01	240.3
.1148E-05	.65	.2125E-04	.7311E+02	.1125E+03	.2079E+02	.2910E+02	43.5

Table 7. (Continued)

.1148E-05	.65	.3628E+02	.3467E-02	.7249E-02	.2501E-01	.2502E-01	240.3
.3750E-06	.60	.7517E-05	.7207E+02	.1201E+03	.2079E+02	.2910E+02	41.8
.3750E-06	.60	.3628E+02	.2428E-02	.5569E-02	.1764E-01	.1765E-01	240.3
.1018E-06	.55	.2227E-05	.7103E+02	.1292E+03	.2079E+02	.2910E+02	40.0
.1018E-06	.55	.3628E+02	.1688E-02	.4260E-02	.1283E-01	.1284E-01	240.3
.2180E-07	.50	.5243E-06	.6999E+02	.1400E+03	.2079E+02	.2910E+02	38.1
.2180E-07	.50	.3628E+02	.1151E-02	.3210E-02	.9454E-02	.9456E-02	240.3
.3405E-08	.45	.9101E-07	.6896E+02	.1532E+03	.2079E+02	.2910E+02	36.2
.3405E-08	.45	.3628E+02	.7607E-03	.2357E-02	.6898E-02	.6899E-02	240.3
.3463E-09	.40	.1041E-07	.6792E+02	.1698E+03	.2079E+02	.2910E+02	34.1
.3463E-09	.40	.3628E+02	.4840E-03	.1671E-02	.4892E-02	.4893E-02	240.3
.1917E-10	.35	.6588E-09	.6688E+02	.1911E+03	.2079E+02	.2910E+02	31.9
.1917E-10	.35	.3628E+02	.2955E-03	.1130E-02	.3319E-02	.3319E-02	240.3
.4293E-12	.30	.1721E-10	.6584E+02	.2195E+03	.2079E+02	.2910E+02	29.5
.4293E-12	.30	.3628E+02	.1748E-03	.7190E-03	.2118E-02	.2118E-02	240.3
.2287E-14	.25	.1100E-12	.6480E+02	.2592E+03	.2079E+02	.2910E+02	27.0
.2287E-14	.25	.3628E+02	.1039E-03	.4202E-03	.1243E-02	.1243E-02	240.3
.1009E-17	.20	.6066E-16	.6376E+02	.3188E+03	.2079E+02	.2910E+02	24.1
.1009E-17	.20	.3628E+02	.6696E-04	.2170E-03	.6446E-03	.6447E-03	240.3
.3181E-23	.15	.2551E-21	.6272E+02	.4181E+03	.2079E+02	.2910E+02	20.9
.3181E-23	.15	.3628E+02	.5082E-04	.9239E-04	.2751E-03	.2751E-03	240.3
.4839E-34	.10	.5821E-32	.6168E+02	.6168E+03	.2079E+02	.2910E+02	17.1
.4839E-34	.10	.3628E+02	.4464E-04	.2757E-04	.8244E-04	.8244E-04	240.3

Table 8. Properties of Superfluid Helium.

1. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.16	.36916E+02	.13784E+02	.60442E+01	.47138E+02	.47350E+02	233.09
2.00	.36804E+02	.92334E+01	.38383E+01	.20947E+02	.20987E+02	237.85
1.95	.36781E+02	.82447E+01	.33344E+01	.18197E+02	.18227E+02	238.65
1.90	.36762E+02	.74296E+01	.29137E+01	.15573E+02	.15596E+02	239.27
1.85	.36744E+02	.67125E+01	.25300E+01	.13443E+02	.13460E+02	239.76
1.80	.36729E+02	.60850E+01	.21863E+01	.11807E+02	.11821E+02	240.14
1.75	.36716E+02	.55369E+01	.18776E+01	.10270E+02	.10280E+02	240.45
1.70	.36705E+02	.50546E+01	.15973E+01	.88219E+01	.88297E+01	240.71
1.65	.36695E+02	.46488E+01	.13554E+01	.74637E+01	.74696E+01	240.94
1.60	.36686E+02	.43126E+01	.11484E+01	.62609E+01	.62652E+01	241.14
1.55	.36679E+02	.40315E+01	.97026E+00	.53194E+01	.53225E+01	241.33
1.50	.36673E+02	.37884E+01	.81092E+00	.45794E+01	.45816E+01	241.51
1.45	.36668E+02	.35743E+01	.66568E+00	.39210E+01	.39225E+01	241.70
1.40	.36663E+02	.33916E+01	.53716E+00	.32723E+01	.32733E+01	241.88
1.35	.36660E+02	.32452E+01	.43093E+00	.26610E+01	.26616E+01	242.08
1.30	.36658E+02	.31295E+01	.34387E+00	.21325E+01	.21328E+01	242.27
1.25	.36656E+02	.30350E+01	.26952E+00	.17083E+01	.17084E+01	242.47
1.20	.36654E+02	.29593E+01	.20776E+00	.13608E+01	.13608E+01	242.67
1.15	.36684E+02	.28983E+01	.15756E+00	.10610E+01	.10610E+01	241.44
1.10	.36684E+02	.28524E+01	.11676E+00	.80340E+00	.80340E+00	241.67
1.05	.36684E+02	.28185E+01	.85279E-01	.59168E+00	.59168E+00	241.90
1.00	.36684E+02	.27941E+01	.61507E-01	.42254E+00	.42254E+00	242.10
.95	.36684E+02	.27771E+01	.44109E-01	.28050E+00	.28050E+00	242.25
.90	.36684E+02	.27654E+01	.31526E-01	.19003E+00	.19004E+00	242.36
.85	.36685E+02	.27576E+01	.22621E-01	.12659E+00	.12660E+00	242.43
.80	.36685E+02	.27524E+01	.16325E-01	.84241E-01	.84254E-01	242.49
.75	.36696E+02	.27473E+01	.11933E-01	.54126E-01	.54138E-01	247.74
.70	.36697E+02	.27451E+01	.89057E-02	.35443E-01	.35453E-01	247.77
.65	.36697E+02	.27437E+01	.67429E-02	.23909E-01	.23916E-01	247.78
.60	.36697E+02	.27427E+01	.51472E-02	.16650E-01	.16655E-01	247.79
.55	.36697E+02	.27420E+01	.39187E-02	.11987E-01	.11990E-01	247.80
.50	.36698E+02	.27415E+01	.29415E-02	.87629E-02	.87650E-02	247.80
.45	.36698E+02	.27411E+01	.21536E-02	.63590E-02	.63602E-02	247.80
.40	.36698E+02	.27409E+01	.15224E-02	.44903E-02	.44910E-02	247.80
.35	.36698E+02	.27407E+01	.10272E-02	.30354E-02	.30357E-02	247.79
.30	.36698E+02	.27406E+01	.65160E-03	.19305E-02	.19307E-02	247.79
.25	.36698E+02	.27405E+01	.37975E-03	.11285E-02	.11285E-02	247.79
.20	.36698E+02	.27405E+01	.19550E-03	.58332E-03	.58333E-03	247.79
.15	.36698E+02	.27405E+01	.83009E-04	.24795E-03	.24795E-03	247.79
.10	.36698E+02	.27405E+01	.24711E-04	.73982E-04	.73982E-04	247.79

2. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE•K	CP	SOUND SPEED METERS/SECOND
2.15	.37369E+02	.16013E+02	.58774E+01	.38667E+02	.38902E+02	232.63
2.00	.37244E+02	.12013E+02	.38774E+01	.21156E+02	.21215E+02	239.83
1.95	.37216E+02	.11013E+02	.33681E+01	.18363E+02	.18406E+02	241.47
1.90	.37193E+02	.10189E+02	.29429E+01	.15711E+02	.15743E+02	242.87
1.85	.37172E+02	.94651E+01	.25553E+01	.13559E+02	.13584E+02	244.06
1.80	.37155E+02	.88316E+01	.22083E+01	.11908E+02	.11927E+02	245.06
1.75	.37139E+02	.82784E+01	.18968E+01	.10358E+02	.10372E+02	245.91
1.70	.37126E+02	.77918E+01	.16140E+01	.88984E+01	.89091E+01	246.63
1.65	.37115E+02	.73821E+01	.13698E+01	.75308E+01	.75388E+01	247.23
1.60	.37105E+02	.70426E+01	.11608E+01	.63198E+01	.63257E+01	247.75
1.55	.37096E+02	.67587E+01	.98082E+00	.53711E+01	.53754E+01	248.18
1.50	.37089E+02	.65130E+01	.81983E+00	.46248E+01	.46279E+01	248.55
1.45	.37083E+02	.62968E+01	.67311E+00	.39608E+01	.39630E+01	248.88
1.40	.37078E+02	.61122E+01	.54325E+00	.33071E+01	.33087E+01	249.16
1.35	.37074E+02	.59642E+01	.43581E+00	.26914E+01	.26924E+01	249.41
1.30	.37071E+02	.58470E+01	.34767E+00	.21590E+01	.21596E+01	249.64
1.25	.37068E+02	.57512E+01	.27233E+00	.17313E+01	.17316E+01	249.86
1.20	.37066E+02	.56745E+01	.20969E+00	.13807E+01	.13809E+01	250.06
1.15	.37098E+02	.56106E+01	.15914E+00	.10747E+01	.10748E+01	250.04
1.10	.37097E+02	.55641E+01	.11778E+00	.81470E+00	.81477E+00	250.23
1.05	.37096E+02	.55297E+01	.85827E-01	.60070E+00	.60073E+00	250.40
1.00	.37096E+02	.55049E+01	.61667E-01	.42940E+00	.42941E+00	250.55
.95	.37095E+02	.54876E+01	.43970E-01	.28536E+00	.28536E+00	250.67
.90	.37095E+02	.54758E+01	.31174E-01	.19312E+00	.19312E+00	250.76
.85	.37096E+02	.54678E+01	.22137E-01	.12819E+00	.12820E+00	250.82
.80	.37096E+02	.54626E+01	.15780E-01	.84734E-01	.84741E-01	250.87
.75	.37092E+02	.54573E+01	.11396E-01	.53810E-01	.53818E-01	254.82
.70	.37092E+02	.54551E+01	.84063E-02	.34735E-01	.34743E-01	254.85
.65	.37093E+02	.54537E+01	.63028E-02	.23055E-01	.23061E-01	254.87
.60	.37093E+02	.54528E+01	.47759E-02	.15803E-01	.15807E-01	254.88
.55	.37093E+02	.54521E+01	.36174E-02	.11232E-01	.11235E-01	254.88
.50	.37093E+02	.54516E+01	.27060E-02	.81390E-02	.81407E-02	254.89
.45	.37093E+02	.54513E+01	.19764E-02	.58736E-02	.58746E-02	254.88
.40	.37093E+02	.54511E+01	.13945E-02	.41334E-02	.41339E-02	254.88
.35	.37093E+02	.54509E+01	.93933E-03	.27877E-02	.27879E-02	254.88
.30	.37094E+02	.54508E+01	.59476E-03	.17695E-02	.17696E-02	254.88
.25	.37094E+02	.54508E+01	.34588E-03	.10321E-02	.10321E-02	254.88
.20	.37094E+02	.54507E+01	.17760E-03	.53195E-03	.53195E-03	254.88
.15	.37094E+02	.54507E+01	.75221E-04	.22524E-03	.22524E-03	254.88
.10	.37094E+02	.54507E+01	.22363E-04	.66931E-04	.66931E-04	254.87

Table 8. (Continued)

4. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEE METERS/SECON
2.13	.38244E+02	.20861E+02	.56718E+01	.31043E+02	.31451E+02	241.72
2.00	.38085E+02	.17533E+02	.39826E+01	.21684E+02	.21826E+02	249.79
1.95	.38043E+02	.16502E+02	.34577E+01	.18786E+02	.18890E+02	252.08
1.90	.38007E+02	.15654E+02	.30199E+01	.16064E+02	.16142E+02	254.06
1.85	.37977E+02	.14910E+02	.26217E+01	.13860E+02	.13919E+02	255.78
1.80	.37950E+02	.14260E+02	.22656E+01	.12168E+02	.12211E+02	257.25
1.75	.37928E+02	.13693E+02	.19462E+01	.10583E+02	.10615E+02	258.52
1.70	.37908E+02	.13194E+02	.16565E+01	.90943E+01	.91183E+01	259.61
1.65	.37892E+02	.12774E+02	.14063E+01	.77010E+01	.77187E+01	260.54
1.60	.37878E+02	.12426E+02	.11920E+01	.64674E+01	.64802E+01	261.32
1.55	.37866E+02	.12135E+02	.10074E+01	.54986E+01	.55079E+01	261.96
1.50	.37855E+02	.11883E+02	.84228E+00	.47345E+01	.47412E+01	262.49
1.45	.37847E+02	.11662E+02	.69196E+00	.40548E+01	.40595E+01	262.93
1.40	.37840E+02	.11473E+02	.55895E+00	.33874E+01	.33907E+01	263.30
1.35	.37834E+02	.11321E+02	.44874E+00	.27596E+01	.27618E+01	263.60
1.30	.37829E+02	.11200E+02	.35817E+00	.22167E+01	.22183E+01	263.86
1.25	.37825E+02	.11102E+02	.28071E+00	.17801E+01	.17811E+01	264.08
1.20	.37822E+02	.11023E+02	.21621E+00	.14219E+01	.14226E+01	264.28
1.15	.37850E+02	.10953E+02	.16471E+00	.11025E+01	.11030E+01	265.31
1.10	.37848E+02	.10905E+02	.12212E+00	.84151E+00	.84181E+00	265.44
1.05	.37846E+02	.10869E+02	.88954E-01	.62520E+00	.62536E+00	265.54
1.00	.37845E+02	.10844E+02	.63674E-01	.45046E+00	.45054E+00	265.63
.95	.37844E+02	.10825E+02	.45002E-01	.30216E+00	.30219E+00	265.70
.90	.37844E+02	.10813E+02	.31424E-01	.20520E+00	.20520E+00	265.75
.85	.37844E+02	.10804E+02	.21836E-01	.13549E+00	.13549E+00	265.80
.80	.37844E+02	.10799E+02	.15175E-01	.87621E-01	.87622E-01	265.83
.75	.37823E+02	.10795E+02	.10650E-01	.54844E-01	.54847E-01	268.03
.70	.37823E+02	.10793E+02	.76436E-02	.34351E-01	.34356E-01	268.06
.65	.37823E+02	.10792E+02	.55978E-02	.21989E-01	.21993E-01	268.08
.60	.37824E+02	.10791E+02	.41674E-02	.14521E-01	.14524E-01	268.09
.55	.37824E+02	.10790E+02	.31197E-02	.10002E-01	.10004E-01	268.10
.50	.37824E+02	.10790E+02	.23173E-02	.70922E-02	.70934E-02	268.10
.45	.37824E+02	.10790E+02	.16859E-02	.50551E-02	.50558E-02	268.10
.40	.37824E+02	.10789E+02	.11869E-02	.35363E-02	.35366E-02	268.10
.35	.37824E+02	.10789E+02	.79811E-03	.23792E-02	.23794E-02	268.10
.30	.37824E+02	.10789E+02	.50433E-03	.15084E-02	.15084E-02	268.10
.25	.37824E+02	.10789E+02	.29242E-03	.87810E-03	.87813E-03	268.09
.20	.37824E+02	.10789E+02	.14950E-03	.45076E-03	.45076E-03	268.09
.15	.37824E+02	.10789E+02	.63056E-04	.18951E-03	.18952E-03	268.09
.10	.37824E+02	.10789E+02	.18749E-04	.55869E-04	.55869E-04	268.09

3. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.14	.37816E+02	.18384E+02	.57551E+01	.33558E+02	.33863E+02	235.85
2.00	.37673E+02	.14777E+02	.39249E+01	.21398E+02	.21491E+02	244.02
1.95	.37638E+02	.13764E+02	.34086E+01	.18556E+02	.18625E+02	246.13
1.90	.37609E+02	.12929E+02	.29778E+01	.15872E+02	.15923E+02	247.96
1.85	.37584E+02	.12196E+02	.25855E+01	.13696E+02	.13735E+02	249.53
1.80	.37562E+02	.11555E+02	.22344E+01	.12026E+02	.12055E+02	250.87
1.75	.37543E+02	.10996E+02	.19193E+01	.10460E+02	.10482E+02	252.02
1.70	.37527E+02	.10504E+02	.16335E+01	.89874E+01	.90035E+01	253.00
1.65	.37513E+02	.10090E+02	.13866E+01	.76082E+01	.76202E+01	253.82
1.60	.37501E+02	.97463E+01	.11752E+01	.63871E+01	.63959E+01	254.52
1.55	.37491E+02	.94590E+01	.99305E+00	.54295E+01	.54359E+01	255.10
1.50	.37483E+02	.92105E+01	.83020E+00	.46754E+01	.46800E+01	255.57
1.45	.37476E+02	.89918E+01	.68182E+00	.40044E+01	.40077E+01	255.97
1.40	.37469E+02	.88052E+01	.55049E+00	.33448E+01	.33470E+01	256.31
1.35	.37464E+02	.86553E+01	.44175E+00	.27237E+01	.27252E+01	256.60
1.30	.37460E+02	.85367E+01	.35246E+00	.21866E+01	.21877E+01	256.85
1.25	.37457E+02	.84396E+01	.27610E+00	.17549E+01	.17556E+01	257.07
1.20	.37454E+02	.83616E+01	.21255E+00	.14010E+01	.14014E+01	257.27
1.15	.37485E+02	.82949E+01	.16160E+00	.10879E+01	.10882E+01	257.96
1.10	.37484E+02	.82477E+01	.11965E+00	.82735E+00	.82753E+00	258.11
1.05	.37482E+02	.82127E+01	.87133E-01	.61211E+00	.61221E+00	258.25
1.00	.37481E+02	.81873E+01	.62455E-01	.43907E+00	.43911E+00	258.36
.95	.37481E+02	.81696E+01	.44313E-01	.29294E+00	.29295E+00	258.45
.90	.37481E+02	.81574E+01	.31168E-01	.19845E+00	.19845E+00	258.52
.85	.37481E+02	.81493E+01	.21892E-01	.13131E+00	.13131E+00	258.57
.80	.37481E+02	.81439E+01	.15409E-01	.85866E-01	.85869E-01	258.61
.75	.37467E+02	.81393E+01	.10974E-01	.54094E-01	.54099E-01	261.57
.70	.37467E+02	.81371E+01	.79888E-02	.34393E-01	.34399E-01	261.60
.65	.37467E+02	.81358E+01	.59231E-02	.22425E-01	.22429E-01	261.62
.60	.37467E+02	.81348E+01	.44509E-02	.15097E-01	.15101E-01	261.63
.55	.37468E+02	.81342E+01	.33525E-02	.10572E-01	.10575E-01	261.64
.50	.37468E+02	.81338E+01	.24992E-02	.75831E-02	.75846E-02	261.64
.45	.37468E+02	.81336E+01	.18217E-02	.54402E-02	.54410E-02	261.64
.40	.37468E+02	.81333E+01	.12837E-02	.38169E-02	.38173E-02	261.64
.35	.37468E+02	.81332E+01	.86373E-03	.25704E-02	.25706E-02	261.64
.30	.37468E+02	.81331E+01	.54622E-03	.16300E-02	.16301E-02	261.64
.25	.37468E+02	.81331E+01	.31713E-03	.94952E-03	.94955E-03	261.63
.20	.37468E+02	.81330E+01	.16247E-03	.48830E-03	.48830E-03	261.63
.15	.37468E+02	.81330E+01	.68659E-04	.20601E-03	.20601E-03	261.63
.10	.37468E+02	.81330E+01	.20406E-04	.60969E-04	.60969E-04	261.63

6. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEED METERS/SECOND
2.11	.39023E+02	.25953E+02	.55803E+01	.30491E+02	.31156E+02	256.93
2.00	.38852E+02	.23021E+02	.41277E+01	.22415E+02	.22691E+02	263.59
1.95	.38795E+02	.21947E+02	.35806E+01	.19368E+02	.19568E+02	265.80
1.90	.38747E+02	.21065E+02	.31249E+01	.16547E+02	.16695E+02	267.68
1.85	.38706E+02	.20292E+02	.27119E+01	.14272E+02	.14382E+02	269.30
1.80	.38671E+02	.19619E+02	.23431E+01	.12523E+02	.12605E+02	270.69
1.75	.38641E+02	.19032E+02	.20126E+01	.10892E+02	.10953E+02	271.90
1.70	.38616E+02	.18517E+02	.17134E+01	.93634E+01	.94081E+01	272.96
1.65	.38594E+02	.18084E+02	.14548E+01	.79346E+01	.79672E+01	273.88
1.60	.38575E+02	.17724E+02	.12332E+01	.66690E+01	.66926E+01	274.66
1.55	.38560E+02	.17422E+02	.10422E+01	.56714E+01	.56882E+01	275.31
1.50	.38546E+02	.17162E+02	.87155E+00	.48813E+01	.48932E+01	275.85
1.45	.38535E+02	.16933E+02	.71644E+00	.41783E+01	.41866E+01	276.29
1.40	.38526E+02	.16738E+02	.57931E+00	.34903E+01	.34961E+01	276.66
1.35	.38519E+02	.16582E+02	.46560E+00	.28444E+01	.28484E+01	276.96
1.30	.38512E+02	.16457E+02	.37206E+00	.22861E+01	.22888E+01	277.21
1.25	.38507E+02	.16356E+02	.29210E+00	.18363E+01	.18381E+01	277.42
1.20	.38503E+02	.16274E+02	.22550E+00	.14673E+01	.14685E+01	277.60
1.15	.38526E+02	.16199E+02	.17237E+00	.11386E+01	.11394E+01	278.66
1.10	.38523E+02	.16149E+02	.12822E+00	.87465E+00	.87517E+00	278.76
1.05	.38521E+02	.16112E+02	.93583E-01	.65498E+00	.65528E+00	278.83
1.00	.38519E+02	.16084E+02	.66941E-01	.47623E+00	.47639E+00	278.88
.95	.38518E+02	.16065E+02	.47062E-01	.32323E+00	.32330E+00	278.92
.90	.38517E+02	.16051E+02	.32484E-01	.22106E+00	.22108E+00	278.95
.85	.38517E+02	.16042E+02	.22147E-01	.14588E+00	.14589E+00	278.98
.80	.38517E+02	.16036E+02	.15021E-01	.92703E-01	.92703E-01	279.00
.75	.38488E+02	.16036E+02	.10239E-01	.57341E-01	.57342E-01	280.18
.70	.38488E+02	.16034E+02	.71325E-02	.34953E-01	.34955E-01	280.20
.65	.38488E+02	.16033E+02	.50842E-02	.21590E-01	.21593E-01	280.22
.60	.38488E+02	.16032E+02	.37060E-02	.13698E-01	.13700E-01	280.23
.55	.38488E+02	.16031E+02	.27352E-02	.90977E-02	.90990E-02	280.24
.50	.38489E+02	.16031E+02	.20154E-02	.62831E-02	.62840E-02	280.24
.45	.38489E+02	.16031E+02	.14607E-02	.44117E-02	.44121E-02	280.24
.40	.38489E+02	.16030E+02	.10269E-02	.30667E-02	.30669E-02	280.24
.35	.38489E+02	.16030E+02	.69015E-03	.20607E-02	.20609E-02	280.24
.30	.38489E+02	.16030E+02	.43574E-03	.13072E-02	.13072E-02	280.24
.25	.38489E+02	.16030E+02	.25216E-03	.76087E-03	.76088E-03	280.24
.20	.38489E+02	.16030E+02	.12846E-03	.38950E-03	.38950E-03	280.24
.15	.38489E+02	.16030E+02	.54003E-04	.16272E-03	.16272E-03	280.24
.10	.38489E+02	.16030E+02	.16096E-04	.47624E-04	.47624E-04	280.24

10. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE·K	CP	SOUND SPEEI METERS/SECONI
2.06	.40313E+02	.35772E+02	.54414E+01	.31007E+02	.32182E+02	285.22
2.00	.40180E+02	.33914E+02	.45147E+01	.24677E+02	.25332E+02	289.49
1.95	.40099E+02	.32711E+02	.39028E+01	.21072E+02	.21518E+02	291.74
1.90	.40032E+02	.31734E+02	.33976E+01	.17896E+02	.18212E+02	293.47
1.85	.39976E+02	.30887E+02	.29448E+01	.15381E+02	.15611E+02	294.80
1.80	.39928E+02	.30153E+02	.25430E+01	.13460E+02	.13630E+02	295.87
1.75	.39887E+02	.29517E+02	.21843E+01	.11698E+02	.11824E+02	296.79
1.70	.39852E+02	.28959E+02	.18605E+01	.10062E+02	.10155E+02	297.60
1.65	.39821E+02	.28489E+02	.15803E+01	.85417E+01	.86103E+01	298.32
1.60	.39796E+02	.28098E+02	.13398E+01	.71950E+01	.72449E+01	298.94
1.55	.39774E+02	.27771E+02	.11321E+01	.61238E+01	.61596E+01	299.47
1.50	.39756E+02	.27488E+02	.94696E+00	.52664E+01	.52917E+01	299.92
1.45	.39741E+02	.27241E+02	.77930E+00	.45019E+01	.45195E+01	300.30
1.40	.39728E+02	.27031E+02	.63143E+00	.37583E+01	.37704E+01	300.63
1.35	.39717E+02	.26862E+02	.50863E+00	.30629E+01	.30712E+01	300.91
1.30	.39709E+02	.26727E+02	.40751E+00	.24612E+01	.24668E+01	301.14
1.25	.39702E+02	.26618E+02	.32130E+00	.19745E+01	.19783E+01	301.32
1.20	.39696E+02	.26530E+02	.24963E+00	.15748E+01	.15774E+01	301.47
1.15	.39713E+02	.26443E+02	.19165E+00	.12396E+01	.12413E+01	301.53
1.10	.39710E+02	.26388E+02	.14334E+00	.95978E+00	.96077E+00	301.60
1.05	.39707E+02	.26347E+02	.10506E+00	.72620E+00	.72676E+00	301.65
1.00	.39705E+02	.26316E+02	.75246E-01	.53481E+00	.53510E+00	301.68
.95	.39703E+02	.26294E+02	.52667E-01	.36996E+00	.37010E+00	301.69
.90	.39702E+02	.26279E+02	.35872E-01	.25644E+00	.25650E+00	301.70
.85	.39701E+02	.26268E+02	.23827E-01	.17036E+00	.17038E+00	301.71
.80	.39701E+02	.26261E+02	.15537E-01	.10686E+00	.10687E+00	301.72
.75	.39667E+02	.26269E+02	.10067E-01	.64699E-01	.64699E-01	301.97
.70	.39667E+02	.26267E+02	.66195E-02	.37923E-01	.37923E-01	301.98
.65	.39667E+02	.26265E+02	.44538E-02	.22084E-01	.22084E-01	301.99
.60	.39667E+02	.26264E+02	.30912E-02	.13000E-01	.13001E-01	302.01
.55	.39667E+02	.26264E+02	.22035E-02	.79908E-02	.79914E-02	302.01
.50	.39667E+02	.26264E+02	.15908E-02	.51870E-02	.51874E-02	302.02
.45	.39667E+02	.26263E+02	.11421E-02	.35083E-02	.35085E-02	302.02
.40	.39667E+02	.26263E+02	.80050E-03	.24003E-02	.24004E-02	302.02
.35	.39667E+02	.26263E+02	.53772E-03	.16083E-02	.16083E-02	302.02
.30	.39667E+02	.26263E+02	.33920E-03	.10219E-02	.10219E-02	302.02
.25	.39667E+02	.26263E+02	.19574E-03	.59501E-03	.59502E-03	302.02
.20	.39667E+02	.26263E+02	.99195E-04	.30328E-03	.30328E-03	302.02
.15	.39667E+02	.26263E+02	.41529E-04	.12544E-03	.12544E-03	302.01
.10	.39667E+02	.26263E+02	.12451E-04	.36373E-04	.36373E-04	302.01

12. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE*K	CP	SOUND SPEED METERS/SECOND
2.04	.40865E+02	.40506E+02	.53391E+01	.30103E+02	.31499E+02	295.59
2.00	.40775E+02	.39333E+02	.47542E+01	.26324E+02	.27274E+02	298.68
1.95	.40679E+02	.38040E+02	.40959E+01	.22242E+02	.22856E+02	301.60
1.90	.40603E+02	.36998E+02	.35578E+01	.18770E+02	.19191E+02	303.71
1.85	.40539E+02	.36105E+02	.30802E+01	.16064E+02	.16364E+02	305.26
1.80	.40486E+02	.35335E+02	.26585E+01	.14016E+02	.14235E+02	306.42
1.75	.40440E+02	.34669E+02	.22834E+01	.12164E+02	.12326E+02	307.39
1.70	.40401E+02	.34088E+02	.19456E+01	.10461E+02	.10581E+02	308.21
1.65	.40368E+02	.33597E+02	.16532E+01	.88876E+01	.89759E+01	308.93
1.60	.40340E+02	.33189E+02	.14018E+01	.74954E+01	.75599E+01	309.56
1.55	.40315E+02	.32846E+02	.11847E+01	.63837E+01	.64303E+01	310.08
1.50	.40295E+02	.32551E+02	.99118E+00	.54894E+01	.55224E+01	310.52
1.45	.40278E+02	.32293E+02	.81621E+00	.46908E+01	.47140E+01	310.90
1.40	.40264E+02	.32074E+02	.66204E+00	.39158E+01	.39318E+01	311.23
1.35	.40252E+02	.31897E+02	.53388E+00	.31920E+01	.32029E+01	311.52
1.30	.40242E+02	.31757E+02	.42827E+00	.25650E+01	.25724E+01	311.75
1.25	.40234E+02	.31643E+02	.33837E+00	.20563E+01	.20612E+01	311.94
1.20	.40228E+02	.31551E+02	.26371E+00	.16380E+01	.16414E+01	312.09
1.15	.40246E+02	.31458E+02	.20286E+00	.13019E+01	.13040E+01	311.59
1.10	.40241E+02	.31401E+02	.15202E+00	.10110E+01	.10123E+01	311.67
1.05	.40238E+02	.31357E+02	.11158E+00	.76749E+00	.76821E+00	311.72
1.00	.40236E+02	.31325E+02	.79964E-01	.56738E+00	.56776E+00	311.75
.95	.40234E+02	.31301E+02	.55912E-01	.39499E+00	.39517E+00	311.76
.90	.40233E+02	.31284E+02	.37943E-01	.27500E+00	.27508E+00	311.77
.85	.40232E+02	.31273E+02	.25001E-01	.18334E+00	.18337E+00	311.78
.80	.40232E+02	.31266E+02	.16075E-01	.11502E+00	.11502E+00	311.79
.75	.40197E+02	.31277E+02	.10202E-01	.69128E-01	.69129E-01	311.86
.70	.40197E+02	.31275E+02	.65370E-02	.40004E-01	.40004E-01	311.87
.65	.40197E+02	.31273E+02	.42744E-02	.22772E-01	.22772E-01	311.88
.60	.40197E+02	.31272E+02	.28894E-02	.12974E-01	.12975E-01	311.89
.55	.40197E+02	.31272E+02	.20189E-02	.76778E-02	.76782E-02	311.90
.50	.40197E+02	.31272E+02	.14400E-02	.48196E-02	.48199E-02	311.90
.45	.40197E+02	.31271E+02	.10278E-02	.31905E-02	.31907E-02	311.90
.40	.40198E+02	.31271E+02	.71893E-03	.21624E-02	.21625E-02	311.90
.35	.40198E+02	.31271E+02	.48263E-03	.14457E-02	.14457E-02	311.90
.30	.40198E+02	.31271E+02	.30426E-03	.91863E-03	.91865E-03	311.90
.25	.40198E+02	.31271E+02	.17537E-03	.53456E-03	.53457E-03	311.90
.20	.40198E+02	.31271E+02	.88722E-04	.27188E-03	.27188E-03	311.90
.15	.40198E+02	.31271E+02	.37127E-04	.11214E-03	.11214E-03	311.90
.10	.40198E+02	.31271E+02	.11148E-04	.32511E-04	.32511E-04	311.90

14. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE•K	CP	SOUND SPEED METERS/SECOND
2.01	.41378E+02	.45148E+02	.52267E+01	.29756E+02	.31369E+02	303.18
2.00	.41346E+02	.44765E+02	.50345E+01	.28446E+02	.29861E+02	304.46
1.95	.41232E+02	.43349E+02	.43147E+01	.23691E+02	.24551E+02	308.74
1.90	.41143E+02	.42229E+02	.37353E+01	.19811E+02	.20375E+02	311.71
1.85	.41071E+02	.41280E+02	.32279E+01	.16849E+02	.17239E+02	313.81
1.80	.41011E+02	.40468E+02	.27835E+01	.14634E+02	.14913E+02	315.34
1.75	.40961E+02	.39770E+02	.23902E+01	.12670E+02	.12874E+02	316.55
1.70	.40918E+02	.39162E+02	.20371E+01	.10888E+02	.11038E+02	317.55
1.65	.40881E+02	.38650E+02	.17316E+01	.92541E+01	.93647E+01	318.40
1.60	.40850E+02	.38223E+02	.14688E+01	.78131E+01	.78941E+01	319.11
1.55	.40823E+02	.37864E+02	.12415E+01	.66592E+01	.67179E+01	319.70
1.50	.40801E+02	.37555E+02	.10391E+01	.57268E+01	.57688E+01	320.19
1.45	.40782E+02	.37286E+02	.85631E+00	.48932E+01	.49228E+01	320.60
1.40	.40766E+02	.37057E+02	.69534E+00	.40859E+01	.41063E+01	320.96
1.35	.40753E+02	.36872E+02	.56136E+00	.33323E+01	.33463E+01	321.28
1.30	.40742E+02	.36725E+02	.45085E+00	.26786E+01	.26880E+01	321.54
1.25	.40733E+02	.36606E+02	.35689E+00	.21463E+01	.21527E+01	321.74
1.20	.40726E+02	.36510E+02	.27891E+00	.17080E+01	.17123E+01	321.91
1.15	.40746E+02	.36410E+02	.21505E+00	.13700E+01	.13727E+01	320.98
1.10	.40741E+02	.36350E+02	.16143E+00	.10672E+01	.10689E+01	321.06
1.05	.40738E+02	.36304E+02	.11862E+00	.81266E+00	.81358E+00	321.12
1.00	.40735E+02	.36269E+02	.85046E-01	.60252E+00	.60300E+00	321.16
.95	.40733E+02	.36244E+02	.59418E-01	.42146E+00	.42169E+00	321.18
.90	.40732E+02	.36227E+02	.40219E-01	.29426E+00	.29436E+00	321.19
.85	.40731E+02	.36214E+02	.26352E-01	.19672E+00	.19676E+00	321.20
.80	.40731E+02	.36207E+02	.16766E-01	.12363E+00	.12364E+00	321.21
.75	.40696E+02	.36222E+02	.10452E-01	.74012E-01	.74013E-01	321.19
.70	.40696E+02	.36219E+02	.65420E-02	.42438E-01	.42438E-01	321.20
.65	.40696E+02	.36218E+02	.41606E-02	.23703E-01	.23703E-01	321.21
.60	.40696E+02	.36217E+02	.27370E-02	.13113E-01	.13113E-01	321.22
.55	.40696E+02	.36216E+02	.18718E-02	.74792E-02	.74795E-02	321.23
.50	.40696E+02	.36216E+02	.13173E-02	.45354E-02	.45356E-02	321.23
.45	.40696E+02	.36216E+02	.93420E-03	.29341E-02	.29342E-02	321.24
.40	.40696E+02	.36216E+02	.65185E-03	.19684E-02	.19685E-02	321.24
.35	.40696E+02	.36216E+02	.43719E-03	.13126E-02	.13126E-02	321.23
.30	.40696E+02	.36216E+02	.27537E-03	.83345E-03	.83346E-03	321.23
.25	.40696E+02	.36216E+02	.15855E-03	.48426E-03	.48426E-03	321.23
.20	.40696E+02	.36216E+02	.80156E-04	.24575E-03	.24575E-03	321.23
.15	.40696E+02	.36216E+02	.33575E-04	.10131E-03	.10131E-03	321.23
.10	.40696E+02	.36216E+02	.10076E-04	.29496E-04	.29496E-04	321.23

16. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE.K	CP	SOUND SPEED METERS/SECOND
1.98	.41864E+02	.49718E+02	.51062E+01	.29285E+02	.31137E+02	308.48
1.00	.41207E+02	.41156E+02	.90552E-01	.64049E+00	.64110E+00	330.02
.95	.41205E+02	.41130E+02	.63229E-01	.44974E+00	.45004E+00	330.05
.90	.41203E+02	.41111E+02	.42724E-01	.31454E+00	.31467E+00	330.07
.85	.41202E+02	.41098E+02	.27886E-01	.21069E+00	.21074E+00	330.09
.80	.41202E+02	.41089E+02	.17608E-01	.13277E+00	.13278E+00	330.10
.75	.41167E+02	.41109E+02	.10815E-01	.79423E-01	.79427E-01	330.05
.70	.41167E+02	.41105E+02	.66270E-02	.45261E-01	.45262E-01	330.06
.65	.41167E+02	.41104E+02	.41032E-02	.24887E-01	.24887E-01	330.07
.60	.41167E+02	.41103E+02	.26250E-02	.13408E-01	.13408E-01	330.08
.55	.41167E+02	.41102E+02	.17543E-02	.73795E-02	.73797E-02	330.08
.50	.41167E+02	.41102E+02	.12165E-02	.43181E-02	.43183E-02	330.09
.45	.41167E+02	.41102E+02	.85646E-03	.27254E-02	.27255E-02	330.09
.40	.41167E+02	.41102E+02	.59592E-03	.18082E-02	.18083E-02	330.09
.35	.41167E+02	.41102E+02	.39917E-03	.12020E-02	.12020E-02	330.09
.30	.41167E+02	.41102E+02	.25114E-03	.76212E-03	.76213E-03	330.09
.25	.41167E+02	.41102E+02	.14449E-03	.44171E-03	.44171E-03	330.09
.20	.41167E+02	.41102E+02	.73081E-04	.22367E-03	.22367E-03	330.09
.15	.41167E+02	.41102E+02	.30697E-04	.92414E-04	.92414E-04	330.09
.10	.41167E+02	.41102E+02	.91828E-05	.27170E-04	.27170E-04	330.09

18. BAR ISOBAR

TEMP K	DENSITY MOLES/LITER	ENTHALPY JOULES/MOLE	ENTROPY	CV JOULES/MOLE•K	CP	SOUND SPEED METERS/SECOND
1.96	.42330E+02	.54239E+02	.49873E+01	.28661E+02	.30808E+02	312.02
1.00	.41654E+02	.45990E+02	.96570E-01	.68147E+00	.68226E+00	338.43
.95	.41652E+02	.45961E+02	.67417E-01	.48020E+00	.48058E+00	338.48
.90	.41651E+02	.45941E+02	.45510E-01	.33624E+00	.33641E+00	338.51
.85	.41650E+02	.45927E+02	.29635E-01	.22558E+00	.22565E+00	338.53
.80	.41649E+02	.45918E+02	.18616E-01	.14262E+00	.14265E+00	338.55
.75	.41614E+02	.45941E+02	.11297E-01	.85490E-01	.85496E-01	338.48
.70	.41614E+02	.45938E+02	.67927E-02	.48552E-01	.48553E-01	338.49
.65	.41614E+02	.45936E+02	.40989E-02	.26361E-01	.26361E-01	338.50
.60	.41614E+02	.45935E+02	.25484E-02	.13869E-01	.13869E-01	338.51
.55	.41614E+02	.45934E+02	.16612E-02	.73748E-02	.73749E-02	338.52
.50	.41614E+02	.45934E+02	.11330E-02	.41587E-02	.41588E-02	338.52
.45	.41614E+02	.45934E+02	.79109E-03	.25550E-02	.25550E-02	338.52
.40	.41614E+02	.45934E+02	.54860E-03	.16743E-02	.16744E-02	338.52
.35	.41614E+02	.45934E+02	.36688E-03	.11089E-02	.11089E-02	338.52
.30	.41614E+02	.45934E+02	.23053E-03	.70140E-03	.70141E-03	338.52
.25	.41614E+02	.45934E+02	.13258E-03	.40507E-03	.40507E-03	338.52
.20	.41614E+02	.45934E+02	.67188E-04	.20470E-03	.20470E-03	338.52
.15	.41614E+02	.45934E+02	.28361E-04	.85065E-04	.85065E-04	338.52
.10	.41614E+02	.45934E+02	.84294E-05	.25414E-04	.25414E-04	338.52

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      FUNCTION HE2(D,TT)
C     MAIN PROPERTIES ROUTINE, FOR THE THERMODYNAMIC PROPERTIES
C     OF HELIUM II.
C     INPUT IS DENSITY IN MOLES/LITER, AND TEMPERATURE IN KELVIN
C     OUTPUT IS PRESSURE, DPDT, DPDD, ENTROPY, ENTHALPY
C     OR CV.  OUTPUT UNITS ARE ATMOSPHERES,MOLES/LITER-KELVIN
C     AND JOULES.
C
      DIMENSION G(8),P(69),PR(23)
      COMMON/DATA 2/PR,R,F
      COMMON/DATA 3/G,RR
      DATA(NR=0),(IH=0)
    1 IF(NR.GT.0)GO TO 2
      CALL DATA HE2
      ND=47
    2 T=TT
      IF(T.LT..1)T=.1
      IF(T.GT.1.199)GO TO 3
      ND=24
      IF(T.GT..799)GO TO 4
      ND=1
      GO TO 4
    3 ND=47
    4 IF(NR.NE.ND)GO TO 5
      GO TO 7
    5 NR=ND
      NF=NR+22
      J=0
      DO 6 I=NR,NF
      J=J+1
    6 PR(J)=P(I)
    7 CONTINUE
      GO TO (11,12,13,14,15,16),M
      ENTRY PHE2
      M=1
      GO TO 1
      ENTRY DPDHE2
      M=2
      GO TO 1
      ENTRY DPTHE2
      M=3
      GO TO 1
      ENTRY SHE2
      M=4
      GO TO 1
      ENTRY HHE2
      IH=1
      M=5
      GO TO 1
      ENTRY CVHE2
      M=6
      GO TO 1
   11 CALL PRESS2(P1,D,T)
      IF(NR.LT.24)GO TO 111
      CALL PRESS3(P3,D,T)
      HE2=P1+P3
      IF(IH.EQ.1)GO TO 17
      RETURN
  111 HE2=P00(D)+P1
      IF(IH.EQ.1)GO TO 17
      RETURN
   12 CALL DPDD2(P1,D,T)
      IF(NR.LT.24)GO TO 112
      CALL DPDD3(P3,D,T)
```

```
      HE2=P1+P3
      RETURN
  112 HE2=P1+DP00(D)
      RETURN
   13 CALL DPDT2(P1,D,T)
      IF(NR.LT.24)GO TO 113
      CALL DPDT3(P3,D,T)
      HE2=P1+P3
      RETURN
  113 HE2=P1
      IF(TT.LT..1)HE2=HE2*TT*10.
      RETURN
   14 DS=SATL2(T)
      CALL DSDN2(P1S,DS,T)
      CALL DSDN2(P1,D,T)
      IF(NR.LT.24)GO TO114
      CALL DSDN3(P3S,DS,T)
      CALL DSDN3(P3,D,T)
      HE2=(P1-P1S+P3-P3S+DVPHE2(T)*(1./D-1./DS))*101.325+S0(T)
      RETURN
  114 HE2=S0(T)+(P1-P1S)*101.325
      IF(TT.LT..1)HE2=HE2*TT*10.
      RETURN
   15 DS=SATL2(T)
      GO TO 11
   17 CONTINUE
      CALL DSDN2(P1S,DS,T)
      CALL PID2(P2S,DS,T)
      CALL DSDN2(P1,D,T)
      CALL PID2(P2,D,T)
      IF(NR.LT.24)GO TO 115
      CALL DSDN3(P3S,DS,T)
      CALL PID3(P4S,DS,T)
      CALL DSDN3(P3,D,T)
      CALL PID3(P4,D,T)
      HE2=((P1-P1S+P3-P3S+DVPHE2(T)*(1./D-1./DS))*T+HE2/D
     1+P2-P2S+P4-P4S)*101.325+H0(T)
      IH=0
      RETURN
  115 HE2=H0(T)+((P1-P1S)*T+(P2-P2S))*101.325+(P00H(D)+HE2/D)*101.325
      HE2=HE2-P00H(DS)*101.325
      IH=0
      RETURN
   16 DS=SATL2(T)
      CALL DSDT2(P1S,DS,T)
      CALL DSDT2(P1,D,T)
      IF(NR.LT.24)GO TO 116
      CALL DSDT3(P2S,DS,T)
      CALL DSDT3(P2,D,T)
      HE2=(P1-P1S+P2-P2S)*101.325+C0(T)+CVVP(D,T)
      RETURN
  116 HE2=(P1-P1S)*101.325+C0(T)
      IF(TT.LT..1)HE2=HE2*TT*10.
   99 CONTINUE
      RETURN
      END
      SUBROUTINE DATA HE2
C     SETS UP THE CONSTANTS FOR THE EQUATION OF STATE
      DIMENSION G(8),P(69),PR(23)
      COMMON/DATA 2/PR,R,P
      COMMON/DATA 3/G,RR
      DATA(A0=2.241456)
      DATA(A1=.1757482)
      DATA(A2=.00470035)
```

```
R=.08205616
RR=R
P( 1)=  -.776003592103E-04
P( 2)=   .516985343553E-04
P( 3)=  -.185460414352E-05
P( 4)=   .993150555179E-06
P( 5)=  -.372729528003E-05
P( 6)=   .905240314118E-04
P( 7)=  -.263138088468E-03
P( 8)=   .210133644446E-03
P( 9)=  -.251675888508E-06
P(10)=   .246805662352E-05
P(11)=  -.235766906295E-04
P(12)=   .636877273619E-04
P(13)=  -.464000281660E-04
P(14)=   .133504455025E-07
P(15)=   .441252121325E-06
P(16)=  -.390205136440E-05
P(17)=   .569946840678E-05
P(18)=  -.219762939629E-05
P(19)=  -.581270462264E-09
P(20)=  -.191711245461E-07
P(21)=   .144897551106E-06
P(22)=  -.793219612515E-07
P(23)=  -.390940913081E-07
P(24)=   .108660418499E-02
P(25)=  -.217871751436E-02
P(26)=   .102911648479E-02
P(27)=   .189253572751E-02
P(28)=  -.674364748289E-02
P(29)=   .798926642309E-02
P(30)=  -.344107467055E-02
P(31)=   .299781633163E-03
P(32)=  -.214084674667E-03
P(33)=   .335439600940E-03
P(34)=   .559092792724E-03
P(35)=  -.119903558078E-02
P(36)=   .526331681180E-03
P(37)=   .118775501632E-04
P(38)=  -.459408808154E-04
P(39)=   .519701003921E-04
P(40)=  -.196070771338E-04
P(41)=   .731453369826E-06
P(42)=  -.526985760908E-06
P(43)=   .169561251135E-05
P(44)=  -.131795348291E-05
P(45)=  -.714287537326E-07
P(46)=   .258759130915E-06
P(47)=  -.299775895293E-03
P(48)=   .261528116001E-03
P(49)=  -.451073420829E-04
P(50)=  -.179926805218E-03
P(51)=   .268760818966E-03
P(52)=   .153832317516E-04
P(53)=  -.162726148595E-03
P(54)=   .477756675722E-04
P(55)=  -.356060361531E-04
P(56)=   .407625370109E-03
P(57)=  -.713769173335E-03
P(58)=   .449456804718E-03
P(59)=  -.913635541095E-04
P(60)=  -.975555037829E-05
P(61)=   .121659779679E-04
P(62)=  -.528306039117E-05
```

```
      P(63)=   .311573112016E-06
      P(64)=   .613299771434E-07
      P(65)=   .506000325098E-06
      P(66)=  -.612590386700E-06
      P(67)=   .230922759488E-06
      P(68)=   .327499222785E-08
      P(69)=  -.515534867647E-08
      G( 1)=  -.320783527549E+01
      G( 2)=   .580145141306E+01
      G( 3)=  -.294344361744E+01
      G( 4)=   .290449403103E-01
      G( 5)=   .801446582474E-01
      G( 6)=  -.175703015761E+00
      G( 7)=   .400129303603E+00
      G( 8)=  -.255176262894E+00
      RETURN
      END
      SUBROUTINE HE2E(P,Q,T)
C     CALCULATE ALL PROPERTIES OF THE F SUB S PART OF THE EQUATION
C     OF STATE, UNITS ARE THE SAME AS IN FUNCTION HE2
      DIMENSION G(8),A(8)
      COMMON/DATA3/G,R
      DATA(A0=2.241456)
      DATA(A1=.1757482)
      DATA(A2=.00470035)
    1 N=1
      IE=1
      Q2=Q*Q
      Q3=Q2*Q
      D0=SATL2(T)
      D02=D0*D0
      D03=D02*D0
      DD0=DSATL2(T)
      DD02=DD0*DD0
      D20=D2SATL(T)
      D=Q
      T2=T*T
      T3=T2*T
      T4=T3*T
      T5=T4*T
      T6=T5*T
      D=D-SATL2(T)
      D2=D**2
      D3=D2*D
      D4=D3*D
      GO TO (10,20,30,40,50,60),II
      ENTRY PRESS3
      II=1
      GO TO 1
   10 A(1)=A2*D3*T3
      A(2)=A2*D3*T**2.5
      A(3)=A2*D3*T2
      A(4)=A1*D2*T3
      A(5)=A1*D2*T2
      A(6)=A0*D*T3
      A(7)=A0*D*T**2.5
      A(8)=A0*D*T2
    2 SUM=0
      DO 3 I=1,8
    3 SUM=SUM+A(I)*G(I)
      P=SUM+P0(Q,T)+VPNHE2(T)
      RETURN
      ENTRY DSDN3
      II=2
```

```
      GO TO 1
   20 A(1)=-A2*(Q2*T2*3/2.-3*Q*(DD0*T3+D0*T2*3)+3*(2*D0*DD0*T3
     1 +3*D02*T2)*ALOG(Q)+(3*D02*DD0*T3+D03*T2*3)/Q)
      A(2)=-A2*(Q2*T**1.5*2.5/2.-3*Q*(DD0*T**2.5+D0*T**1.5*2.5)
     1 +3*ALOG(Q)*(2*D0*DD0*T**2.5+2.5*D02*T**1.5)
     2 +(3.*D02*DD0*T**2.5+2.5*D03*T**1.5)/Q)
      A(3)=-A2*(Q2*T-3*Q*(DD0*T2+D0*T*2)+3*ALOG(Q)*(2*D0*DD0*T2
     1 +2*D02*T)+(3*D02*DD0*T2+2*D03*T)/Q)
      A(4)=-A1*(3*T2*Q-ALOG(Q)*(6*T2*D0+2*T3*DD0)
     1 -(3*T2*D02+T3*DD0*D0*2)/Q)
      A(5)=-A1*(T*Q*2-ALOG(Q)*(4*T*D0+2*T2*DD0)
     1 -(2*T*D02+T2*DD0*D0*2)/Q)
      A(6)=-A0*(3*ALOG(Q)*T2 +(DD0*T3+T2*D0*3)/Q)
      A(7)=-A0*(2.5*ALOG(Q)*T**1.5+(DD0*T**2.5+2.5*D0*T**1.5)/Q)
      A(8)=-A0*(2*ALOG(Q)*T+(DD0*T2+T*D0*2)/Q)
      SUM=-A0*DD0/Q+A1*2*DD0*ALOG(Q)+2*A1*D0*DD0/Q
     1 -6*A2*D0*DD0*ALOG(Q)-3*A2*D02*DD0/Q+3*A2*DD0*Q
      DO 22 I=1,8
   22 SUM=SUM+A(I)*G(I)
      P=SUM
      RETURN
      ENTRY DSDT3
      II=3
      GO TO 1
   30 A(1)=-A2*(Q2*T*3-3*Q*(D20*T3+DD0*T2*6+D0*T*6)+3*(2*DD02*T3
     1 +2*D0*D20*T3+6*D0*DD0*T2+6*DD0*D0*T2+6*D02*T)*ALOG(Q)
     2 +(3*D0*DD02*T3*2+3*D02*D20*T3+9*D02*DD0*T2+9*D02*DD0*T2
     3 +6*D03*T)/Q)*T
      A(2)=-A2*(Q2*T**.5*3.75/2.-3*Q*(D20*T**2.5+DD0*T**1.5*2.5
     1 +DD0*T**1.5*2.5+D0*T**.5*3.75)+3*ALOG(Q)*(2*DD02*T**2.5
     2 +2*D0*D20*T**2.5+5*D0*DD0*T**1.5+5*DD0*D0*T**1.5+3.75*
     3 D02*T**.5)+(6*DD02*D0*T**2.5+3*D02*D20*T**2.5+7.5*D02*
     4 DD0*T**1.5+7.5*DD0*D02*T**1.5+3.75*D03*T**.5)/Q)*T
      A(3)=-A2*(Q2-3*Q*(D20*T2+DD0*T*4+D0*2)+3*ALOG(Q)*(2*DD02*T2
     1 +D0*D20*T2*2+8*D0*DD0*T+2*D02)+(3*D02*D20*T2+6*DD02*D0*T2
     2 +12*D02*DD0*T+2*D03)/Q)*T
      A(4)=-A1*(6*T*Q-ALOG(Q)*(6*T2*DD0+12*T*D0+6*T2*DD0+2*T3*D20
     1 )-(6*DD0*D0*T2+6*T*D02+6*T2*DD0*D0+2*T3*D20*D0+2*T3*DD02)/Q)*T
      A(5)=-A1*(Q*2-ALOG(Q)*(8*T*DD0+4*D0+2*T2*D20)-(2*D02
     1+8*T*D0*DD0+2*T2*D20*D0+2*T2*DD02)/Q)*T
      A(6)=-A0*(6*ALOG(Q)*T+(D20*T3+3*DD0*T2+6*T*D0+3*T2*DD0)/Q)*T
      A(7)=-A0*(3.75*ALOG(Q)*T**.5+(D20*T**2.5+DD0*T**1.5*2.5
     1 +2.5*DD0*T**1.5+3.75*D0*T**.5)/Q)*T
      A(8)=-A0*(2*ALOG(Q)+(D20*T2+2*DD0*T+2*D0+2*T*DD0)/Q)*T
      SUM=-A0*D20/Q+A1*D20*ALOG(Q)*2+2*A1*(DD02+D0*D20)/Q
     1 -6*A2*ALOG(Q)*(DD02+D0*D20)-3*A2*(D20*D02+2*DD02*D0)/Q
     2 +3*A2*D20*Q
      SUM=SUM*T
      DO 32 I=1,8
   32 SUM=SUM+A(I)*G(I)
      P=SUM
      RETURN
      ENTRY DPDC3
      II=4
      GO TO 1
   40 A(1)=A2*D2*T3*3.
      A(2)=A2*D2*3.*T**2.5
      A(3)=A2*D2*T2*3.
      A(4)=A1*D*T3*2.
      A(5)=A1*D*T2*2.
      A(6)=A0*T3
      A(7)=A0*T**2.5
      A(8)=A0*T2
   41 SUM=0
```

```
      DO 42 I=1,8
   42 SUM=SUM+A(I)*G(I)
      P=SUM+DPO(Q,T)
      RETURN
      ENTRY DPDT3
      II=5
      GO TO 1
   50 A(1)=A2*(3*D2*(-DDO)*T3+3*T2*D3)
      A(2)=A2*(3*D2*(-DDO)*T**2.5+2.5*T**1.5*D3)
      A(3)=A2*(3*D2*(-DDO)*T2+2*T*D3)
      A(4)=A1*(2*D*(-DDO)*T3 + 3*T2*D2)
      A(5)=A1*(2*D*(-DDO)*T2 +2*T*D2)
      A(6)=A0*((-DDO)*T3+3*T2*D  )
      A(7)=A0*((-DDO)*T**2.5+2.5*T**1.5*D)
      A(8)=A0*((-DDO)*T2+2*T*D)
   51 SUM=0
      DO 52 I=1,8
   52 SUM=SUM+A(I)*G(I)
      P=SUM+DVPHE2(T)+DPODT(Q,T)
      RETURN
      ENTRY PID3
C INTEGRATE P/Q**2 WITH RESPECT TO Q
      II=6
      GOTO 1
   60 QINTGR= .5*Q**2-3*Q*DO+3*ALOG(Q)*DO**2+DO**3/Q
      A(1)=A2*T**3.0*QINTGR
      A(2)=A2*T**2.5*QINTGR
      A(3)=A2*T**2.0*QINTGR
      QINTGR=Q-2*ALOG(Q)*DO-DO**2/Q
      A(4)=A1*T**3.0*QINTGR
      A(5)=A1*T**2.0*QINTGR
      QINTGR=ALOG(Q)+DO/Q
      A(6)=A0*T**3.0*QINTGR
      A(7)=A0*T**2.5*QINTGR
      A(8)=A0*T**2.0*QINTGR
      SUM=0
      DO 62 I=1,8
   62 SUM=SUM+A(I)*G(I)
      P=SUM-VPNHE2(T)/Q+A0*ALOG(Q)+A1*Q-A1*2*DO*ALOG(Q)
     A+A0*DO/Q-DO**2*A1/Q+DO**3*A2/Q
     1+A2*Q**2/2-A2*3*DO*Q+A2*3*DO**2*ALOG(Q)
      RETURN
      END
      FUNCTION PO(DI,T)
C     CALCULATE THE PRESSURE FOR THE PO PART OF THE EQUATION OF
C     STATE,UNITS ARE THE SAME AS IN FUNCTION HE2
      DATA(A0=2.241456)
      DATA(A1=.1757482)
      DATA(A2=.00470035)
      X=DI-SATL2(T)
      PO=A0*X+A1*X**2+A2*X**3
      RETURN
      END
      FUNCTION DPODT(DI,T)
C     CALCULATE DPODT, UNITS ARE THE SAME AS IN FUNCTION HE2
      DATA(A0=2.241456)
      DATA(A1=.1757482)
      DATA(A2=.00470035)
      X=DI-SATL2(T)
      DXDT=-DSATL2(T)
      DPODT=A0*DXDT+A1*2*X*DXDT+A2*3*X**2*DXDT
      RETURN
      END
      FUNCTION DPO(DI,T)
```

```
C     CALCULATE DPOCD, UNITS ARE THE SAME AS IN THE FUNCTION HE2
      DATA(A0=2.241456)
      DATA(A1=.1757482)
      DATA(A2=.00470035)
      X=DI-SATL2(T)
      DP0=A0+A1*2.*X+A2*3.*X**2
      RETURN
      END
      FUNCTION SATL2(T)
C     CALCULATE THE SATUARATED LIQUID DENSITY IN MOLES/LITER FOR
C     A TEMPERATURE IN KELVIN
      X=2.172-T
      D=-3.31007-.0074243913*X+.0059164737533*X*ALOG(X)
      D=EXP(D)*1000.
      SATL2=D
      IF(T.LT..8)D=36.27877
      RETURN
      END
      FUNCTION DSATL2(T)
C     1ST DERIVATIVE OF SATL2
      X=2.172-T
      DSATL2=+.00742434913-.0059164737533*(1.+ALOG(X))
      DSATL2=DSATL2*SATL2(T)
      RETURN
      END
      FUNCTION D2SATL(T)
C     2ND DERIVATIVE OF SATL2
      X=2.172-T
      D=SATL2(T)
      D2=.0059164737553*D/X+(.00742434913-.0059164737553*(ALOG(X)+1))
     1*DSATL2(T)
      D2SATL=D2
      RETURN
      END
      FUNCTION CVVP(D,T)
C     CALCULATES THE CONTRIBUTION OF THE VAPOR PRESSURE TERM TO CV
      D0=SATL2(T)
      CVVP=DVPHE2(T+.005)-DVPHE2(T-.005)
      CVVP=CVVP*T*101.325*(1./D-1./D0)/.01
      RETURN
      END
      SUBROUTINE HE2EG(P,Q,T)
C     CALCULATES THE F SUB S TERM IN THE EQUATION OF STATE
C     UNITS ARE THE SAME AS IN THE FUNCTION HE2
      DIMENSION G(23),A(23)
      COMMON/DATA 2/G,R
    1 N=1
      D=Q-36.27877
      T2=T*T
      T3=T2*T
      T4=T3*T
      T5=T4*T
      T6=T5*T
      T7=T6*T
      T8=T6*T2
      T9=T8*T
      T10=T8*T2
      T11=T10*T
      D2=D*D
      D3=D2*D
      D4=D3*D
      D5=D4*D
      D6=D5*D
      GO TO (10,20,30,40,50,60),II
```

```
      ENTRY PRESS2
      II=1
      GO TO 1
   10 A(1)=-D2*T4/4.
      A(2)=-D2*T5/5.
      A(3)=-D2*T6/6.
      A(4)=-D3*T3*2./3.
      A(5)=-D3*T4*2./4.
      A(6)=-2.*D3*T5/5.
      A(7)=-2.*D3*T6/6.
      A(8)=-2.*D3*T7/7.
      A(9)=-3.*D4*T3/3.
      A(10)=-3.*D4*T4/4.
      A(11)=-3.*D4*T5/5.
      A(12)=-3.*D4*T6/6.
      A(13)=-3.*D4*T7/7.
      A(14)=-4.*D5*T3/3.
      A(15)=-4.*D5*T5/5.
      A(16)=-4.*D5*T7/7.
      A(17)=-4.*D5*T9/9.
      A(18)=-4.*D5*T11/11.
      A(19)=-5.*D6*T3/3.
      A(20)=-5.*D6*T5/5.
      A(21)=-5.*D6*T7/7.
      A(22)=-5.*D6*T9/9.
      A(23)=-5.*D6*T11/11.
    2 SUM=0
      DO 3 I=1,23
    3 SUM=SUM+A(I)*G(I)*Q**2/D2
      P=SUM
      RETURN
      ENTRY DSDN2
      II=2
      GO TO 1
   20 A(1)=T3*D
      A(2)=D*T4
      A(3)=D*T5
      A(4)=D2*T2
      A(5)=D2*T3
      A(6)=D2*T4
      A(7)=D2*T5
      A(8)=D2*T6
      A(9)=D3*T2
      A(10)=D3*T3
      A(11)=D3*T4
      A(12)=D3*T5
      A(13)=D3*T6
      A(14)=D4*T2
      A(15)=D4*T4
      A(16)=D4*T6
      A(17)=D4*T8
      A(18)=D4*T10
      A(19)=D5*T2
      A(20)=D5*T4
      A(21)=D5*T6
      A(22)=D5*T8
      A(23)=D5*T10
   21 SUM=0
      DO 22 I=1,23
   22 SUM=SUM+A(I)*G(I)
      P=SUM
      RETURN
      ENTRY DSDT2
      II=3
```

```
      GO TO 1
   30 A(1)=3*T2*D
      A(2)=4*T3*D
      A(3)=5*T4*D
      A(4)=2*D2*T
      A(5)=3*T2*D2
      A(6)=4*T3*D2
      A(7)=5*T4*D2
      A(8)=6*T5*D2
      A(9)=2*T*D3
      A(10)=3*T2*D3
      A(11)=4*T3*D3
      A(12)=5*T4*D3
      A(13)=6*T5*D3
      A(14)=2*T*D4
      A(15)=4*T3*D4
      A(16)=6*T5*D4
      A(17)=8*T7*D4
      A(18)=10*T9*D4
      A(19)=2*T*D5
      A(20)=4*T3*D5
      A(21)=6*T5*D5
      A(22)=8*T7*D5
      A(23)=10*T9*D5
   31 SUM=0
      DO 32 I=1,23
   32 SUM=SUM+A(I)*G(I)
      P=SUM*T
      RETURN
      ENTRY DPDD2
      II=4
      GO TO 1
   40 A(1)=-2.*Q*T4/4.
      A(2)=-2.*Q*T5/5.
      A(3)= -2.*Q*T6/6.
      A(4)=-(Q**2+2*Q*D)*2*T3/3.
      A(5)=-(Q**2+2*Q*D)*2*T4/4.
      A(6)=-(Q**2+2*Q*D)*2*T5/5.
      A(7)=-(Q**2+2*Q*D)*2*T6/6.
      A(8)=-(Q**2+2*Q*D)*2*T7/7.
      A(9)=-(Q**2*2*D+2*Q*D2)*3*T3/3.
      A(10)=-(Q**2*2*D+2*Q*D2)*3*T4/4.
      A(11)=-(Q**2*2*D+2*Q*D2)*3*T5/5.
      A(12)=-(Q**2*2*D+2*Q*D2)*3*T6/6.
      A(13)=-(Q**2*2*D+2*Q*D2)*3*T7/7.
      A(14)=-(Q**2*D2*3+2*Q*D3)*4*T3/3.
      A(15)=-(Q**2*D2*3+2*Q*D3)*4*T5/5.
      A(16)=-(Q**2*D2*3+2*Q*D3)*4*T7/7.
      A(17)=-(Q**2*D2*3+2*Q*D3)*4*T9/9.
      A(18)=-(Q**2*D2*3+2*Q*D3)*4*T11/11.
      A(19)=-(Q**2*D3*4+2*Q*D4)*5*T3/3.
      A(20)=-(Q**2*D3*4+2*Q*D4)*5*T5/5.
      A(21)=-(Q**2*D3*4+2*Q*D4)*5*T7/7.
      A(22)=-(Q**2*D3*4+2*Q*D4)*5*T9/9.
      A(23)=-(Q**2*D3*4+2*Q*D4)*5*T11/11.
   41 SUM=0
      DO 42 I=1,23
   42 SUM=SUM+A(I)*G(I)
      P=SUM
      RETURN
      ENTRY DPDT2
      II=5
      GO TO 1
   50 A(1)=-D2*T3
```

```
      A(2)=-D2*T4
      A(3)=-D2*T5
      A(4)=-2*D3*T2
      A(5)=-2*D3*T3
      A(6)=-2*D3*T4
      A(7)=-2*D3*T5
      A(8)=-2*D3*T6
      A(9)=-3*D4*T2
      A(10)=-3*D4*T3
      A(11)=-3*D4*T4
      A(12)=-3*D4*T5
      A(13)=-3*D4*T6
      A(14)=-4*D5*T2
      A(15)=-4*D5*T4
      A(16)=-4*D5*T6
      A(17)=-4*D5*T8
      A(18)=-4*D5*T10
      A(19)=-5*D6*T2
      A(20)=-5*D6*T4
      A(21)=-5*D6*T6
      A(22)=-5*D6*T8
      A(23)=-5*D6*T10
   51 SUM=0
      DO 52 I=1,23
   52 SUM=SUM+A(I)*G(I)*Q**2/D2
      P=SUM
      RETURN
      ENTRY PID2
      II=6
      GO TO 1
   60 A(1)=-D*T4/4
      A(2)=-D*T5/5
      A(3)=-D*T6/6
      A(4)=-D2*T3/3
      A(5)=-D2*T4/4
      A(6)=-D2*T5/5
      A(7)=-D2*T6/6
      A(8)=-D2*T7/7
      A(9)=-D3*T3/3
      A(10)=-D3*T4/4
      A(11)=-D3*T5/5
      A(12)=-D3*T6/6
      A(13)=-D3*T7/7
      A(14)=-D4*T3/3
      A(15)=-D4*T5/5
      A(16)=-D4*T7/7
      A(17)=-D4*T9/9
      A(18)=-D4*T11/11
      A(19)=-D5*T3/3
      A(20)=-D5*T5/5
      A(21)=-D5*T7/7
      A(22)=-D5*T9/9
      A(23)=-D5*T11/11
   61 SUM=0
      DO 62 I=1,23
   62 SUM=SUM+A(I)*G(I)
      P=SUM
      RETURN
      END
      FUNCTION SO(TT)
C     CALCULATES THE 0 PRESSURE VALUE FOR S,H, AND CV
C     UNITS ARE THE SAME AS IN FUNCTION HE2
      DIMENSION S(44),C(44),H(44),T(44)
      DATA(T=0,.1,.15,.2,.25,.3,.35,.4,.45,.5,.55,.6,.65,.7,.75,.8,
```

```
      1.85,.9,.95,1.,1.05,1.1,1.15,1.2,1.25,1.3,1.35,1.4,1.45,1.5,
     21.55,1.6,1.65,1.7,1.75,1.8,1.85,1.9,1.95,2.,2.05,2.1,2.15,2.172)
      DATA(C=0,.20597E-4,.68827E-4,.16143E-3,.31144E-3,.53134E-3,
     1.83306E-3,.0012284,.0017327,.0023749,.0032236,.0044293,
     2.0062754,.0091734,.01383,.021058,.031759,.047598,.070005,.10504,
     3.1465,.19816,.26074,.33523,.42128,.52642,.65752,.80923,
     4.97015,1.1334,1.3169,1.5506,1.8494,2.1868,2.5464,
     52.9278,3.3332,3.8614,4.5113,5.19,5.9009,6.6428,
     68.298466,14.)
      DATA(S=0,.68889E-5,.23112E-4,.54338E-4,.10534E-3,.18041E-3,
     1.28386E-3,.41983E-3,.59254E-3,.80709E-3,.10714E-2,.0014005,
     2.0018226,.0023865,.0031649,.0042735,.0058517,.0080843,.01123,
     3.015562,.021457,.029233,.039275,.051689,.066910,.085256,.10675,
     4.13301,.16481,.20076,.24020,.28431,.33558,.39554,.46502,
     5.54155,.62676,.72189,.82613,.95098,1.0920,1.2394,1.4137
     679,1.559)
      DATA(H=0,.4174E-5,.6261E-5,.1181E-4,.2337E-4,.4412E-4,.7787E-4,
     1.1290E-3,.2025E-3,.3046E-3,.4436E-3,.6331E-3,.8974E-3,.1278E-2,
     2.1844E-2,.2705E-2,.4008E-2,.5966E-2,.8878E-2,.01311,.01916,
     3.02754,.03883,.05344,.0721,.09543,.124,.1602,.2054,.2584,.3186,
     4.3882,.4715,.5721,.6917,.8276,.9832,1.161,1.363,1.608,1.891,
     52.197,2.525,2.916)
      S0=ATKINT(TT,S,T,44,6,NES,.01)*4.0026
      RETURN
       ENTRY C0
      S0=ATKINT(TT,C,T,44,6,NES,.01)*4.0026
      RETURN
      ENTRY H0
      S0=ATKINT(TT,H,T,44,6,NES,.01)*4.0026
      RETURN
       END
      FUNCTION FINDD2(PI,T)
C     CALCULATES THE DENSITY IN MOLES/LITER FOR AN INPUT
C     OF PRESSURE IN ATMOSPHERES AND TEMPERATURE IN KELVIN.
      P=PI
      IF(PI.LT..00001)P=.00001
      D=SATL2(T)
      D=D+(43.-D)*P/25.
      ID=1
      DO 10 I=1,25
      PP=PHE2(D,T)
      IF(ABS(PP-P).LE..00001*P)GO TO 11
      DP=DPDHE2(D,T)
   10 D=D-(PP-P)/DP
      FINDD2=D
      RETURN
   11 FINDD2=D
      RETURN
      END
      FUNCTION PMELT2(T)
C     INPUT IN KELVIN, OUTPUT IN ATMOSPHERES  (FROM GRILLY, 1972)
      IF(T.LT.1.464)GO TO 1
      PMELT2=31.168-17.122*T+9.292*T**2
      RETURN
    1 PMELT2=24.996+.0799014422*T-.6729427939*T**2+1.87853695*T**3
     1-2.326509762*T**4+1.061136353*T**5
      RETURN
      END
      FUNCTION FIND TD(DD)
C     CALCULATE THE TEMPERATURE IN K ON THE LAMBDA LINE FOR
C     AN INPUT OF DENSITY IN GM/CC  (KIERSTEND 1967)
      D=DD
      T=2.172
      DO 10 I=1,20
```

```
      DC=DENLAM(T)
      IF(ABS(DD-DC).LE..000001*DD)GO TO 11
   10 T=T-(DENLAM(T)-DD)/(DELAMP(T))
      GO TO 12
   11 FIND TD=T
      RETURN
   12 FIND TD=0
      RETURN
      END
      FUNCTION DENLAM(T)
C     CALCULATE THE DENSITY IN GM/CC ON THE LANBDA LINE FOR A
C     TEMPERATURE IN K      (KIERSTEND  1967)
      D0=0.14841388
      D1=-0.150735
      D2=-0.3298225
      D3=-0.53031333
      D4=-0.383035
      D5=-0.00226388
      D6=36.7348
      X=T-2.172
      DENLAM=D0+D1*X+D2*X**2+D3*X**3+D4*X**4+C5*EXP(D6*X)
      RETURN
      END
      FUNCTION DELAMP(T)
C     CALCULATE THE DERIVATIVE OF DENLAM WITH RESPECT TO TEMPERATURE
      D0=0.14841388
      D1=-0.150735
      D2=-0.3298225
      D3=-0.53031333
      D4=-0.383035
      D5=-0.00226388
      D6=36.7348
      X=T-2.172
      DELAMP=D1+2*D2*X+3*D3*X**2+4*D4*X**3+D5*C6*EXP(D6*X)
      RETURN
      END
      FUNCTION FIND TP(PP)
C     CALCULATE THE TEMPERATURE ON THE LAMBDA LINE FOR A GIVEN
C     PRESSURE IN ATM   (KIERSTEND  1967)
      P=PP
      T=2.172
      DO 10 I=1,20
      BC=PRSLAM(T)
      IF (ABS(PP-BC).LE..000001*PP) GO TO 11
   10 T=T-(PRSLAM(T)-PP)/(PRLAMP(T))
      GO TO 12
   11 FIND TP=T
      RETURN
   12 FIND TP=0
      RETURN
      END
      FUNCTION PRSLAM(T)
C     CALCULATE THE PRESSURE IN ATMOSPHERES FOR AN INPUT OF
C     TEMPERATURE IN KELVIN   (KIERSTEND  1967)
      B0=0.42800749
      B1=-95.0719
      B2=-86.417
      B3=-103.341
      B4=-77.52175
      B5=-0.37827065
      B6=42.2507
      X=T-2.172
      PRSLAM=B0+B1*X+B2*X**2+B3*X**3+B4*X**4+B5*EXP(B6*X)
      RETURN
```

```
      END
      FUNCTION PRLAMP (T)
C     CALCULATE THE DERIVATIVE OF PRSLAM
      B0=0.42800749
      B1=-95.0719
      B2=-86.417
      B3=-103.341
      B4=-77.52175
      B5=-0.37827065
      B6=42.2507
      X=T-2.172
      PRLAMP=B1+2*B2*X+3*B3*X**2+4*B4*X**3+B5*B6*EXP(B6*X)
      RETURN
      END
      FUNCTION P00(DI)
C     CALCULATE THE PRESSURE IN ATM AT T=0 FOR AN INPUT OF DENSITY
C     IN MOLES/LITER
      X=DI-36.27877
      P00=2.281877372*X+.16820886*X**2+.005277884968*X**3
      RETURN
      END
      FUNCTION P00H(DI)
C     CALCULATE THE CONTRIBUTION OF P00 TO ENTHALPY
      DATA(A=36.27877),(B=2.281877372),(C=.16820886),(F=.005277884968)
      P00H=B*ALOG(DI)+B*A/DI+C*DI-2*C*A*ALOG(DI)-C*A**2/DI+F*DI**2/2.
     1-3*F*A*DI+3*F*A**2*ALOG(DI)+F*A**3/DI
      RETURN
      END
      FUNCTION DP00(DI)
C     CALCULATE THE DERIVATIVE OF P0 WITH RESPECT TO DENSITY
      X=DI-36.27877
      DP00=2.281877372+.16820886*2.*X+.005277884968*3.*X**2
      RETURN
      END
      FUNCTION ATKINT(X,YMAT,XMAT,NELMTS,NMAX,NESSY,ACRCY)
      DIMENSION YMAT(999), XMAT(999),A(21,20)
C     FIRST TWO SUCCESSIVE VALUES OF THE XMATRIX THAT STRADDLE THE
C     VALUE X WILL BE SOUGHT
      JJ1=NELMTS-1
      DO 20 I=1,JJ1
      DIF1=X-XMAT(I)
      DIF2=XMAT(I+1)-X
      IF(DIF1)16,15,16
   15 ATKINT=YMAT(I)
      NESSY =NMAX
      RETURN
   16 IF(DIF2)18,17,18
   17 ATKINT=YMAT(I+1)
      NESSY =NMAX
      RETURN
   18 RATIO=DIF1/ DIF2
      IF(RATIO)20,20,19
   19 IMID=I
      GO TO 32
   20 CONTINUE
      NESSY=1
      ATKINT=0.0
      RETURN
   32 CONTINUE
C     NOTE THAT RATIO IS POSITIVE IF THE TWO POINTS STRADDLE X
C     REGARDLESS WHICH IS LARGER
      JJJ=IMID
      JUP=IMID
      JDN=IMID
```

```
      IF(JJJ+NMAX-NELMTS+1)98,98,102
   98 DO 201 J=1,NMAX
      JJJ=IMID+J-1
      A(1,J)=XMAT(JJJ)
  201 A(2,J)=YMAT(JJJ)
      GO TO 203
  102 DO 41 J=1,NMAX
      JJ=J/2
      JOE=J-2*JJ
C     JOE IS 0 IF J IS EVEN AND 1 IF J IS ODD
      IF(J-1)33,40,33
   33 IF(JDN-1)34,36,34
   34 IF(JUP-NELMTS)35,37,35
   35 IF(JOE)37,36,37
   36 JUP=JUP+1
      JJJ=JUP
      GO TO 40
   37 JDN=JDN-1
      JJJ=JDN
      GO TO 40
   40 A(1,J)=XMAT(JJJ)
      A(2,J)=YMAT(JJJ)
   41 CONTINUE
  203 NNN=NMAX+1
      DO 6 J=3,NNN
      L=J-1
      DO 5 K=L,NMAX
C     J IS THE COLUMN NUMBER
C     K IS THE ROW NUMBER
     0A(J,K)=(A(J-1,K)-A(J-1,J-2))*(X-A(1,J-2))/(A(1,K)-A(1,J-2))
     1      +A(J-1,J-2)
      IF(K-L)3,2,3
    2 IF(ABS((A(J,L)-A(J-1,L-1))/A(J,L))-ACRCY/100.0)7,7,3
    3 CONTINUE
    5 CONTINUE
    6 CONTINUE
      NESSY=0
      ATKINT=A(NNN,NMAX)
      RETURN
    7 NESSY=J-1
      ATKINT=A(J,L)
      RETURN
      END
      FUNCTION VPNHE2(TT)
C     GIVES A VAPOR PRESSURE FOR HELIUM IN ATMOSPHERES GIVEN A
C     TEMPERATURE IN KELVIN.  THE FUNCTION REPRODUCES THE 1968 HELIUM
C     TEMPERATURE SCALE TO .0001 KELVIN
      DIMENSION C(12),D(14)
      DATA(C=-3.9394635287,141.27497598,-1640.7741565,11974.557102,
     1-55283.309818,166219.56504,-325212.82840,398843.22750,
     2-277718.06992,83395.204183)
      DATA(D=-49.510540356,651.9236417,-3707.5430856,12880.673491,
     1 -30048.545554,49532.267436,-59337.558548,52311.296025,
     2-33950.233134,16028.674003,-5354.1038967,1199.0301906,
     3 -161.46362959,9.8811553386)
      T=TT
      P=0
      IF(T-2.1720)10,10,1
    1 T=T-DELT(T)
      DO 5 I=1,10
    5 P=P+C(I)*T**(2-I)
      VP =EXP(P)/.76E+6
      VPNHE2=VP
      RETURN
```

```
   10 CONTINUE
      T=T-DELT(T)
      IF(T.LT..8)GO TO 20
      DO 15 I=1,14
   15 P=P+D(I)*T**(2-I)
      VP =EXP(P)/.76E+6
      VPNHE2=VP
      RETURN
   20 VPNHE2=VPNLOW(TT)
      RETURN
      END
      FUNCTION DELT(TT)
      T=TT
      DELT=.001+.002*T
      RETURN
      END
      FUNCTION VPTHE2(PP)
C     SOLVES THE VAPOR PRESSURE EQUATION FOR TEMPERATURE GIVEN A PRESSURE
C     THE FLUID IS HELIUM AND THE UNITS ARE ATMOSPHERES AND KELVINS
      P=PP
      IF(P.LT..842105)GO TO 12
      T=5.0
      PCAL=VPNHE2(T)
      GO TO 13
   12 PCAL=.049737
      IF(ABS(P-PCAL)-.0000001*PP)11,11,1
    1 T=2.1720
   13 DO 10 I=1,26
      DP=DVPHE2(T)
      DEL=(PCAL-P)/DP
      T=T-DEL
      PCAL=VPNHE2(T)
      IF(ABS(P-PCAL)-.0000001*P)11,11,2
    2 IF(ABS(DEL)-.0000001*T)11,11,10
   10 CONTINUE
      PRINT 100,T
   11 VPTHE2=T
      RETURN
  100 FORMAT(* TEMPERATURE ITTERATION FAILED AT T=*,E14.7)
      END
      FUNCTION DVPHE2(TT)
C     GIVES THE DERIVATIVE OF THE VAPOR PRESSURE FOR HELIUM GIVEN A
C     TEMPERATURE IN KELVINS  PRESSURE IS IN ATMOSPHERES
      DIMENSION C(12),D(14)
      DATA(C=-3.9394635287,141.27497598,-1640.7741565,11974.557102,
     1-55283.309818,166219.56504,-325212.82840,398843.22750,
     2-277718.06992,83395.204183)
      DATA(D=-49.510540356,651.9236417,-3707.5430856,12880.673491,
     1 -30048.545554,49532.267436,-59337.558548,52311.296025,
     2-33950.233134,16028.674003,-5354.1038967,1199.0301906,
     3 -161.46362959,9.8811553386)
      P=0
      IF(TT-2.1720)10,10,1
    1 T=TT-DELT(TT)
      DO 5 I=1,10
    5 P=P+C(I)*T**(1-I)*(2-I)
      DVPHE2=P*VPNHE2(TT)
      RETURN
   10 IF(TT.LT..8)GO TO 20
      T=TT-DELT(TT)
      DO 15 I=1,14
   15 P=P+D(I)*T**(1-I)*(2-I)
      DVPHE2=P*VPNHE2(TT)
      RETURN
```

```
   20 DVPHE2=DLOW(TT)
      RETURN
      END
      FUNCTION VPNLOW(T)
      R=.08205616
      H=H0(T)/4.0026
      S=S0(T)/4.0026
      GL=H-T*S
      D=.0001
      IF(T.LT..7)D=.0000001
      IF(T.LT..5)D=.1E-8
      IF(T.LT..35)D=1.E-12
      IF(T.LT..3)D=1.E-13
      IF(T.LT..25)D=1.E-16
      IF(T.LT..2)D=1.E-30
      IF(T.LT..15)D=.1E-34
      DO 10 I=1,20
      GV=36.805468+5.193043*(T-4.22-T*ALOG(T/4.22))-T*9.37855
     1+25.31479*T*R*ALOG(R*T*D)
      IF(ABS(GL-GV).LE..001*(ABS(GL)))GO TO 11
      P=D*R*T
      DGV=25.31469*R*T/D
   10 D=D-(GV-GL)/DGV
      PRINT 100
  100 FORMAT(*  NO CONVERGENCE*)
   11 VPNLOW=D*R*T
      RETURN
      END
      FUNCTION DLOW(T)
      TUP=T+.05*T
      TDN=T-.05*T
      PUP=VPNLOW(TUP)
      PDN=VPNLOW(TDN)
      DLOW=(PUP-PDN)/(TUP-TDN)
      RETURN
      END
      FUNCTION ENTR2(D,T)
      DIMENSION A(20),G(20)
      COMMON/DATA2/G,A,F,N,ID
      ID=1
      CALL DSDN2(S1,D,T)
      DD=SATL2(T)
      CALL DSDN2(SO,DD,T)
      S=S0(T)+(DVPHE2(T)*(1./D-1./DD)+(S1-SO))*101.325
      ENTR2=S
      RETURN
      END
      FUNCTION CPHE2(D,T)
C     CALCULATE THE SPECIFIC HEAT CAPACITY AT CONSTANT PRESSURE
C     UNITS ARE THE SAME AS IN FUNCTION HE2
      CPHE2=CVHE2(D,T)+101.325*(DPTHE2(D,T)**2/DPDHE2(D,T)/D**2)
      RETURN
      END
      FUNCTION WHE2(D,T)
C     CALCULATE THE VELOCITY OF SOUND FOR AN INPUT OF DENSITY IN
C     MOLES/LITER
      WHE2=((CPHE2(D,T)/CVHE2(D,T))*DPDHE2(D,T)*25311.)**.5
      RETURN
      END
```

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15. SUPPLEMENTARY NOTES

☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here)

The equation of state of He-II is modeled by an equation of state explicit in pressure as a function of density and temperature. The equation of state is divided into three regions of temperature 0 to .8 K; .8 to 1.2 K and 1.2 to the lambda temperature for which similar functional forms are used with different adjustable parameters. The combined functions are valid over the entire PT range of the superfluid and may be used for all classical thermodynamic properties. Comparisons between calculated and experimental data are presented. A computer program for calculation of thermodynamic properties (PVT, isochoric heat capacity, isobaric heat capacity, internal energy, enthalpy, entropy, and velocity of sound) is included.

17. KEY WORDS (six to twelve entries; alphabetical order; capitalize only the first letter of the first key word unless a proper name; separated by semicolons)

Computer program; equation of state; deviation plots; helium II; mathematical model; superfluid; thermodynamic properties.

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NOTE: The principal publication outlet for the foregoing data is the Journal of Physical and Chemical Reference Data (JPCRD) published quarterly for NBS by the American Chemical Society (ACS) and the American Institute of Physics (AIP). Subscriptions, reprints, and supplements available from ACS, 1155 Sixteenth St., NW, Washington, DC 20056.

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